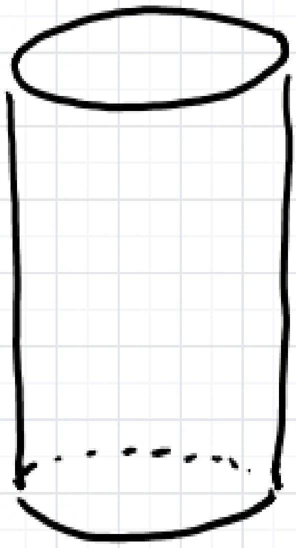
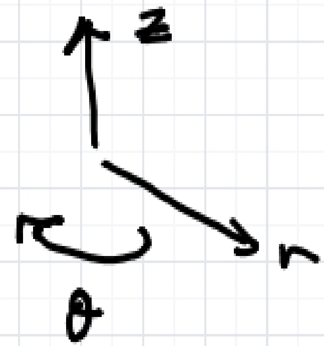
LECTURE # 7

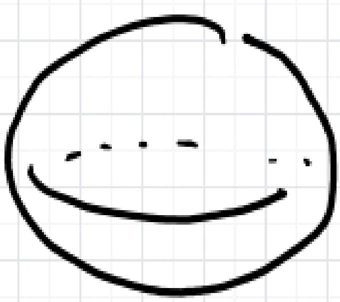
OTHER COORDINATE SYSTEMS (Ch 15.6)



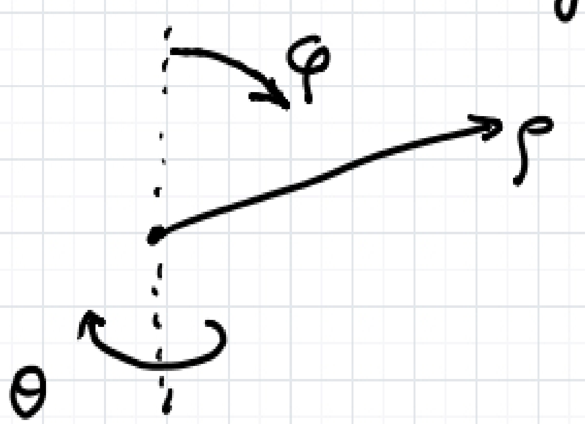
Best described by



Cylindrical Coordinates.



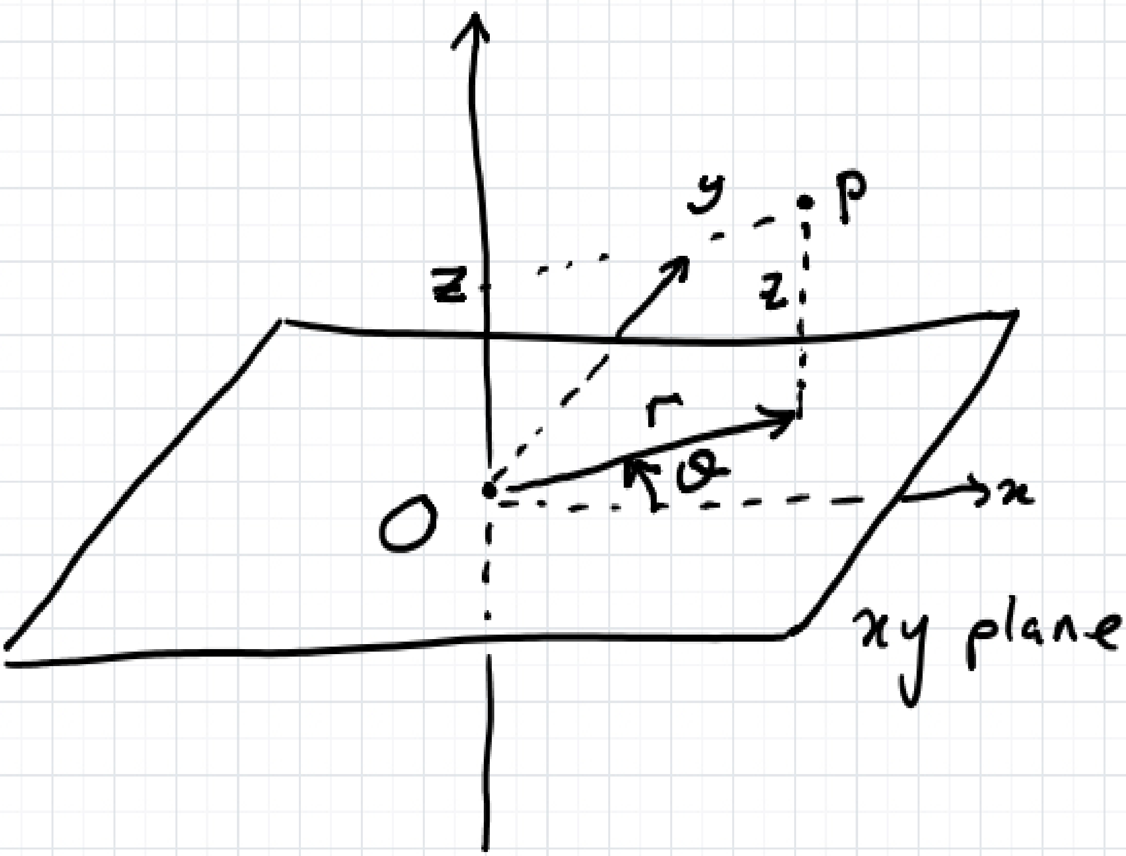
Best described by



Spherical coordinates.

CYLINDRICAL COORDINATES

Extend polar (r,θ) coordinates for xy-plane by coordinate z



<u>Dictionary</u> $(x,y,z) \longleftrightarrow (r,\theta,z)$

$$x = r\cos\theta$$

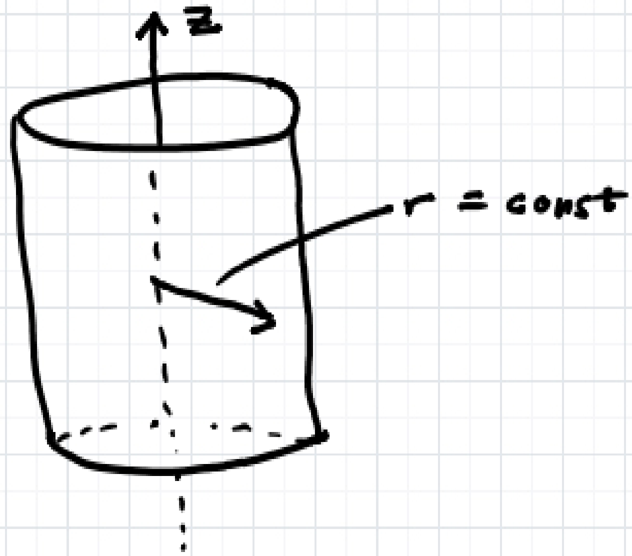
$$y = r\sin\theta$$

$$z = z$$

Surfaces of constant r, θ, z:

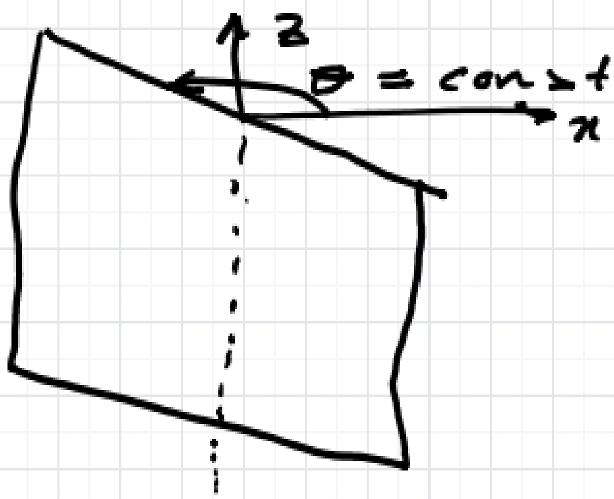
$r = \text{const}$

CYLINDER
ABOUT
z-AXIS



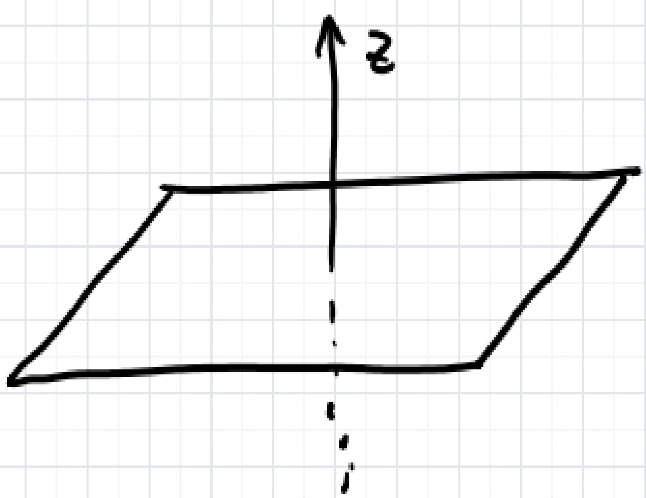
$\theta = \text{const}$

PLANE
THRU'
z-AXIS



$z = \text{const}$

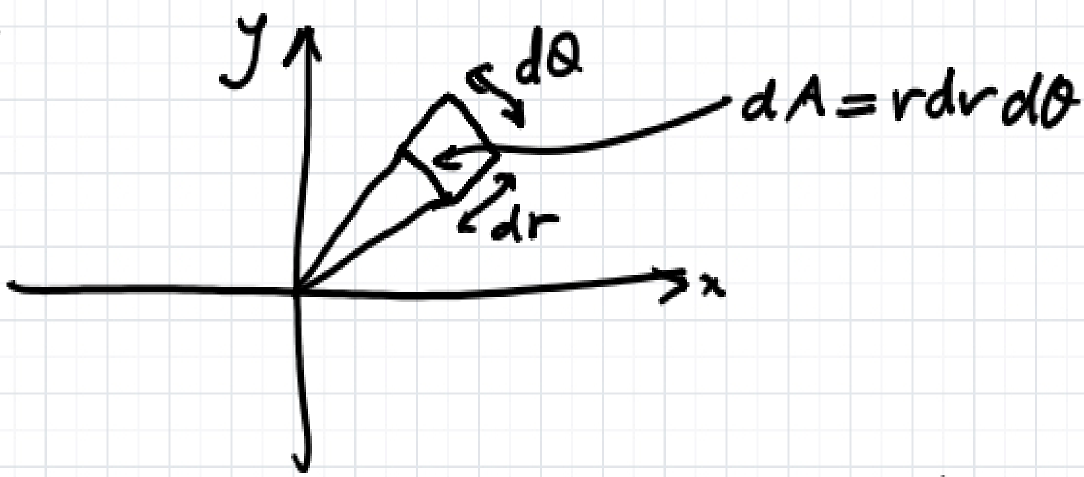
PLANE
$\parallel$ to
xy-PLANE



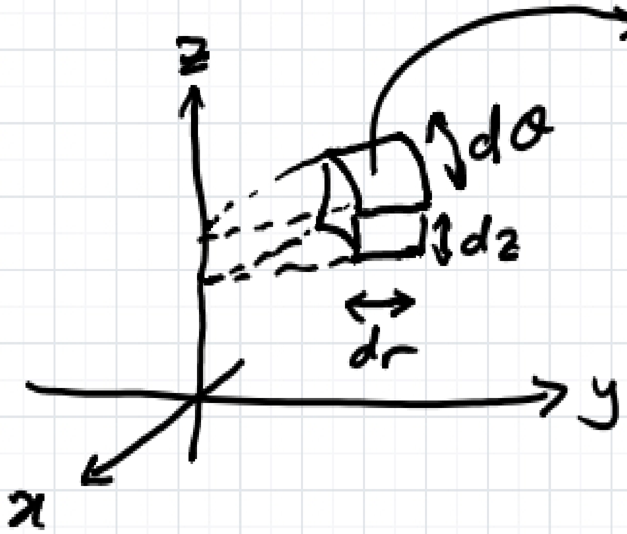
VOLUME ELEMENT

$$\boxed{dV = r\,dr\,d\theta\,dz}$$

Because



So

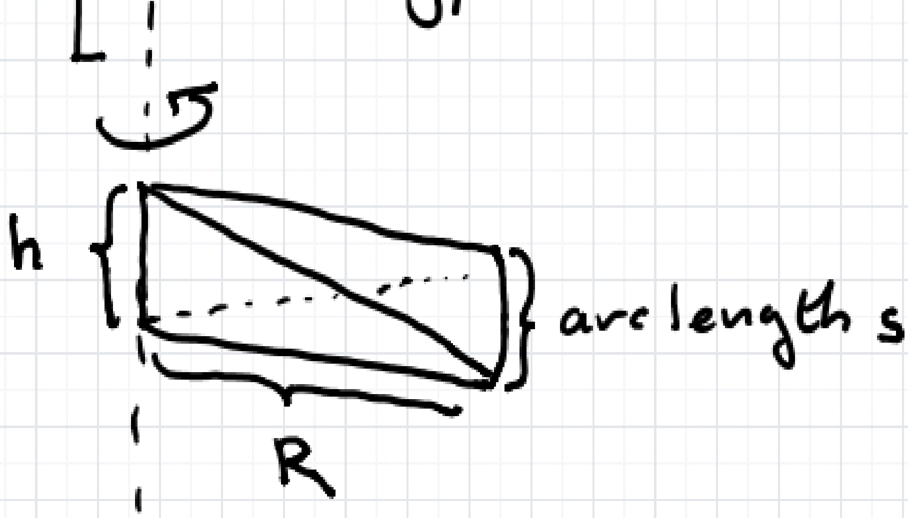


$dV =$ base × height

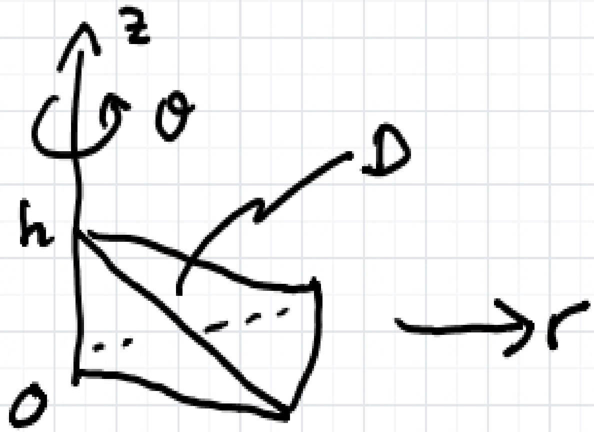
$= dA\,dz$

$= r\,dr\,d\theta\,dz$

<u>Example:</u> Find the moment of inertia of the rotating blade of mass M shown around the axis L (assume constant density)

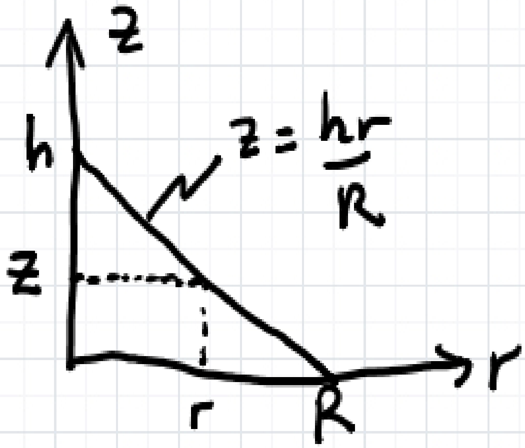


Choose cylindrical coordinates



Here $0 \leq z \leq 1$ $\quad 0 \leq \theta \leq \frac{s}{R}$

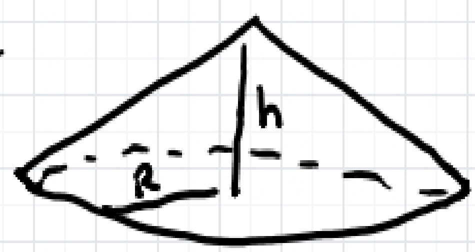
For r limits of integration look at cross section



$$0 \le r \le \frac{Rz}{h}$$

$\Rightarrow$ Do r integral <u>FIRST</u>

Note that the cone



has volume

$$V = \frac{1}{3} h \times \text{base} = \frac{\pi R^2 h}{3}$$

$$\Rightarrow \text{BLADE VOLUME} = \left(\frac{s}{2\pi R}\right) \frac{\pi R^2 h}{3} = \frac{sRh}{6}$$

$$\Rightarrow \text{BLADE DENSITY} = \frac{M}{sRh/6} = \frac{6M}{sRh}$$

Hence

$$I_L = \int_D dV\, r^2 \rho$$

$$= \int_0^{\frac{s}{R}} d\theta \int_0^h dz \int_0^{\frac{Rz}{h}} r^3 dr\, \frac{6M}{sRh}$$

$$= \frac{6M}{sRh} \left(\frac{s}{R}\right) \int_0^h dz\, \frac{1}{4} \left(\frac{Rz}{h}\right)^4$$

$$= \frac{6M}{sRh} \left(\frac{s}{R}\right) \frac{1}{4} \cdot \frac{1}{5} \frac{R^4}{h^4} \cdot h^5$$

$$= \frac{3MR^2}{10}$$ (Classic result - explain)

Integrating functions over volumes

Just as for regions in the plane if $f : D \to \mathbb{R}$ is some function of a domain D we may consider

$$\int_D dV\, f$$

These integrals have applications analogous to area integrals. For example the average value of f over the domain D is

$$\boxed{\bar{f} \equiv \langle f \rangle \equiv \frac{\int_D f\, dV}{\int_D dV} = \frac{\int_D f\, dV}{\text{volume of } D}}$$