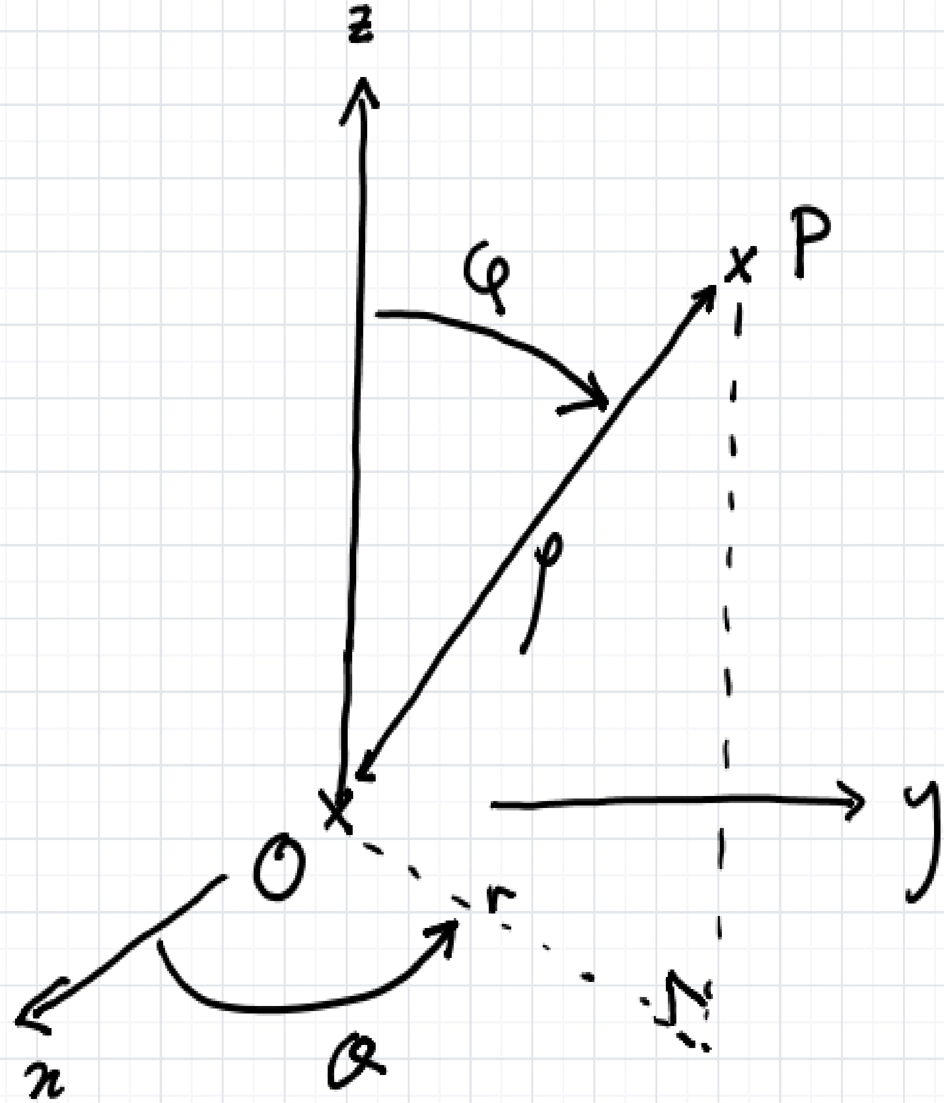
LECTURE #8

SPHERICAL COORDINATES



Dictionary Cylindrical Spherical

$$z = \rho \cos\varphi$$

$$r = \rho \sin\varphi$$

$$\theta = \theta$$

Dictionary   Cartesian   spherical

$$x = \rho \sin\varphi \cos\theta$$

$$y = \rho \sin\varphi \sin\theta$$

$$z = \rho \cos\varphi$$

Nb $x^2+y^2+z^2 = \rho^2 \sin^2\varphi \cos^2\theta + \rho^2 \sin^2\varphi \sin^2\theta + \rho^2 \cos^2\varphi$

$$= \rho^2 \sin^2\varphi \left[\cos^2\theta + \sin^2\theta\right] + \rho^2 \cos^2\varphi$$

$$= \rho[\sin^2\varphi + \cos^2\varphi] = \rho^2$$

FUN REMARK: Note n-dimensional spherical coordinates exist and are given by

$$x_1 = \rho \cos\varphi$$

$$x_2 = \rho \sin\varphi \cos\theta_1$$

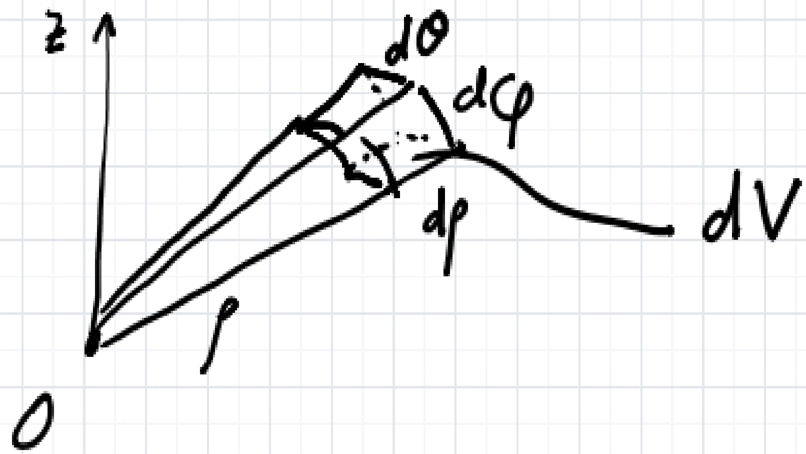
$$x_3 = \rho \sin\varphi \sin\theta_1 \cos\theta_2$$

$$x_4 = \rho \sin\varphi \sin\theta_1 \sin\theta_2 \cos\theta_2$$

$$\vdots$$

& $x_1^2+x_2^2+x_3^2+x_4^2+\dots = \rho^2$

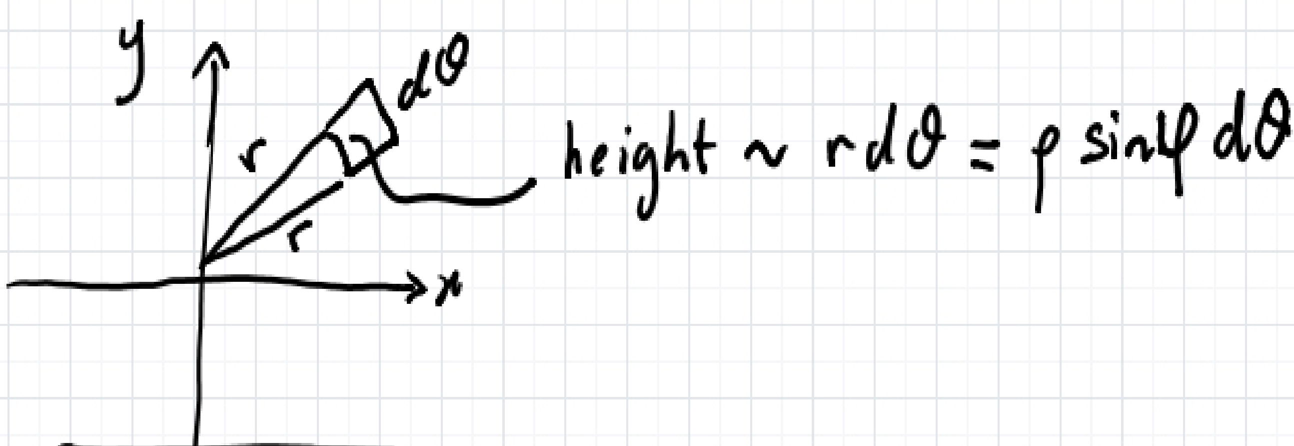
VOLUME ELEMENT IN SPHERICAL COORDINATES:



CROSS SECTION

FROM SIDE

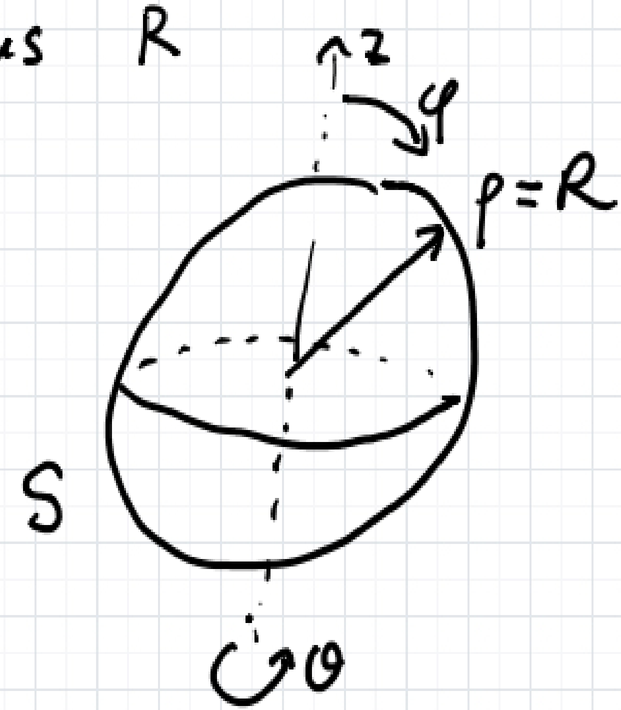
CROSS SECTION FROM ABOVE



$$\Rightarrow \boxed{dV = \rho^2 \sin\varphi \, d\theta \, d\varphi}$$

<u>Example</u> (Easy) Volume of a sphere

radius R



$0 \le \rho \le R$

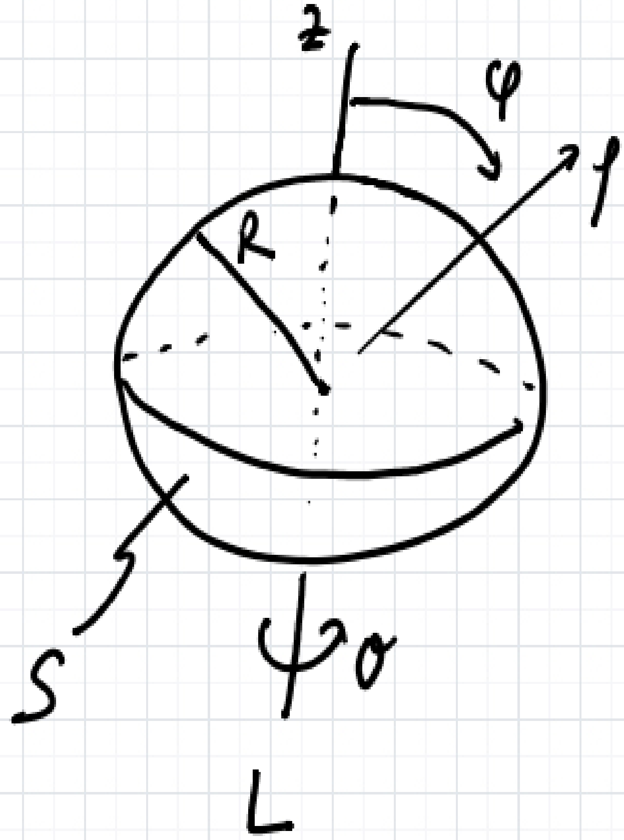
$0 \le \varphi < \pi$

$0 \le \theta < 2\pi$

$$\text{Volume} = \int_S dV = \int_0^{2\pi} d\theta \int_0^R \rho^2 d\rho \int_0^{\pi} d\varphi \sin\varphi$$

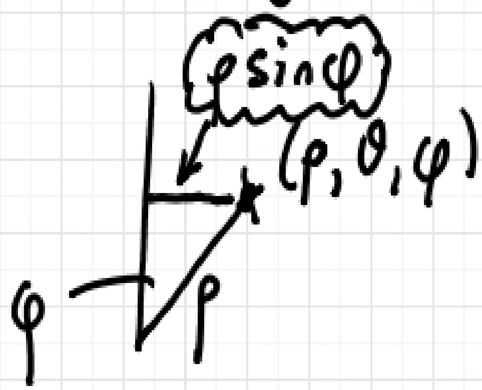
$$= 2\pi \, \frac{1}{3} R^3 \int_{-\cos\varphi = -1}^{-\cos\varphi = +1} d(-\cos\varphi) = \frac{4\pi R^3}{3}$$

Example Moment of inertia of a mass M, radius R sphere about an axis through its center



Density = $\frac{M}{\frac{4}{3}\pi R^3}$

Distance from r axis of rotation



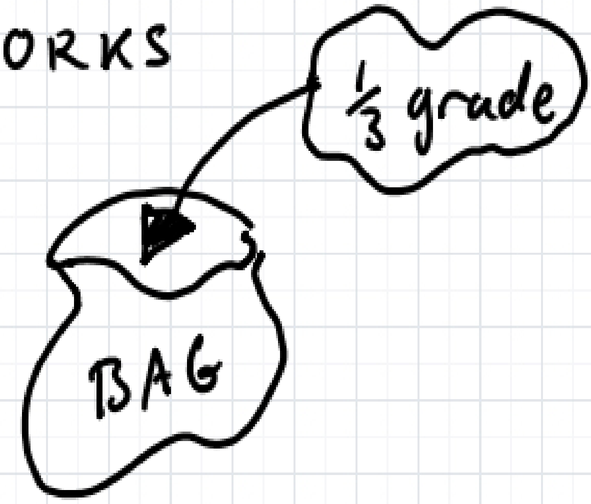
$$I_L = \int_S dV \rho\, r^2 = \int_0^{2\pi} d\theta \int_0^R \rho^2 d\rho \int_0^{\pi} d\varphi \sin\varphi \left[\frac{M}{\frac{4}{3}\pi R^3}\right] \rho^2 \sin^2\varphi$$

$$= \frac{M}{\frac{4}{3}\pi R^3}\, 2\pi\, \frac{1}{5} R^5 \underbrace{\int_0^{\pi} d\varphi \sin^3\varphi}_{4/3} = \frac{2}{5} M R^2$$

MIDTERM I HINTS

1. DO ALL THE SUGGESTED HOMEWORKS

$\Rightarrow$



2. WORK THROUGH LECTURES

3. UNDERSTAND WHAT THE VARIOUS MOMENT INTEGRALS REPRESENT & HOW TO CALCULATE THEM.

Good Luck!

Integrating functions over volumes

Just as for regions in the plane if $f : D \to \mathbb{R}$ is some function of a domain D we may consider

$$\int_D dV\, f$$

These integrals have applications analogous to area integrals. For example the average value of f over the domain D is

$$\boxed{\bar{f} \equiv \langle f \rangle \equiv \frac{\int_D f\, dV}{\int_D dV} = \frac{\int_D f\, dV}{\text{volume of } D}}$$