22A Homework 4

Due Friday April 27, 5pm Wellman Boxes

KOIL $\equiv$ Kolman/Hill, Edition 8, "*Introductory Linear Algebra*"

Question 1 *Elementary Matrices:* Let A be an $m \times n$ matrix. Write down $m \times m$ matrices E_{rs}, $D_r(\lambda)$ and $T_{rs}(\mu)$ such that upon multiplying A on the left

- E_{rs} swaps rows r and s,
- $D_r(\lambda)$ multiplies row r by λ,
- $T_{rs}(\mu)$ adds μ times row s to row r.

Compute the determinant of the three matrices you found.

Question 2 Let

$$A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} .$$

Compute $\det A$. The trace of a matrix (denoted "tr") is defined to be the sum of its diagonal elements (for example $\mathrm{tr}\ A \equiv a + e + i$). Compute $\alpha = \mathrm{tr}A$, $\beta = \mathrm{tr}A^2$ and $\gamma = \mathrm{tr}A^3$. Work out a formula for $\det A$ as a polynomial in α, β and γ [1].

Question 3 In class we defined powers of matrices. The exponential of an $n \times n$ matrix A is defined by a power series

$$\exp(A) \equiv 1 + A + \frac{1}{2!}A^2 + \frac{1}{3!}A^3 + \cdots .$$

Study the special case

$$A_\lambda = \begin{pmatrix} 0 & \lambda \\ 0 & 0 \end{pmatrix}$$

Compute $\exp(A_\lambda)$. Prove $\exp(A_{\lambda+\mu}) = \exp(A_\lambda)\exp(A_\mu)$.

[1] For example, if A is any 2×2 matrix, it is easy to see that $\det A = \frac{1}{2}(\mathrm{tr}A)^2 - \frac{1}{2}\mathrm{tr}A^2$.

Question 4 In 50 words or less evaluate the course to date. Include suggestions for how the presentation could be improved. Comment on how you think you are faring so far!

Question 5 KOIL 1.8, p 113, qq 1, 2, 5, 6.

Question 6 KOIL pp 114-116, qq S2, S8, S16, S18, S28, S30.

Question 7 KOIL pp 116-117, qq T4, T6, T23.

Question 8 KOIL pp 117-118, qq C2, C8.

Question 8 KOIL 3.1, pp 192-194, qq 1, 2, 4, 6abc, 10, 16a, 22, 24.

Question 9 KOIL 3.1, pp 194-195, qq T6, T12, T18

Question 10 Prove $\det(AB) = \det A \det B$ for 2×2 matrices.

Question 11 *Relax!*