22A Homework 4

Due Friday May 4, 5pm Wellman Boxes

KOIL $\equiv$ Kolman/Hill, Edition 8, "*Introductory Linear Algebra*"

Question 1 *The adjoint matrix:* Explain, in your own words[1], why the adjoint matrix $\mathrm{adj}(A)$, built as the transpose of the matrix of cofactors, obeys

$$\mathrm{adj}(A)A = (\det A)I\,.$$

Question 2 *Cramer's rule:* Prove Cramer's rule for a non-singular linear system of two equations in two unknowns.

Question 3 We have already studied triangular matrices, but there is a more general notion of a "block triangular matrix". Prove that 3×3 matrices of shape

$$\begin{pmatrix} \star & \star & \star \\ \star & \star & \star \\ & & \star \end{pmatrix}\,,$$

close[2] under multiplication.

Question 4 KOIL 3.2, pp 207-209, qq 1, 2, 6, 8, 12, 14, 16, 18, 24.

Question 5 KOIL 3.2, pp 209-210, qq T4, T8, T10.

Question 6 KOIL p 213, qq C2, C4, C6.

Question 7 *Parallel transport:* Two explorers, named Christopher Columbus and Christina Columbina, explore respectively a flat and sphere shaped earth. Christopher travels around a large triangular loop. At each point of his journey he carries an arrow (given to him by his friend Cupid) and is very careful to keep the arrow pointing in the same direction at all times. Christina begins her journey at

[1] Read Koil 3.2 carefully, then lock the book in the freezer and proceed...

[2] *I.e.*, the multiplication of a pair of such matrices again gives a matrix of this shape.

the north pole, travels south to the equator, along the equator for several thousand miles, finally returning to the north pole. She also carries an arrow (supplied by Rudolph) being careful not to change its direction, even if she herself turns a corner. Later, at a local coffee shop, they compare their findings, focussing mainly on the arrows. Recount (in 30 words or less) their conversation. Explain, using pictures, if necessary, what happened.

Question 8 KOIL 4.1, pp 227-228, qq 2, 4, 8, 12, 16, 20, 22, 24, 30.

Question 9 KOIL 4.1 p 228, qq T2, T3, T4, T5, T6, T8.

Question 10 Pretend you are a math professor. In 30 words or less, write question 1 on the UC Davis, Spring 2007, 22A Final Exam.