250A Homework 4

Due Monday November 1

Question 1 Find all quotient groups for D_8. Draw the lattice diagram and indicate which subgroups are normal. Also, compute and compare all composition series of D_8. The same for S_4.

Question 2 Show that $H \trianglelefteq G$ and $K \trianglelefteq G \Rightarrow H \cap K \trianglelefteq G$ and $HK \trianglelefteq G$. Suppose the normal subgroups K and H are both maximal in G, what does this imply for their product KH?

Question 3 If $N \trianglelefteq G$ and G/N is an infinite cyclic group, prove that G has a normal subgroup of index n $\forall n \in \mathbb{N}$.

Question 4 Show that Z_8 has Z_2 and Z_4 as homomorphic images. What can you say about the homomorphic images of cyclic groups? What about abelian groups?

Question 5 Dcompose A_4 and A_5 into conjugacy classes. (Hint, what happens to the cyclic form of a permutation under conjugation?)

Question 6 Construct explicit examples of both 1st and 2nd isomorphism theorems.