250A Homework 5

Due Monday November 8

Question 1 Subgroups $H \leq S_4$ act on $S \equiv \{1, 2, 3, 4\}$ in the natural way. For each subgroup H below, compute the orbits and stabilizer of each $s \in S$:

(i) $H = \langle (123) \rangle$

(ii) $H = \langle (1234) \rangle$

(iii) $H = \langle (12), (34) \rangle$

(iv) $H = D_8$

(v) $H = A_4$

Question 2 Let $G = \mathbb{R} \ni r$ be the group of real numbers under addition. Show that

(i) $S = \mathbb{R}^n \ni x \mapsto x + rv$ ($v \in \mathbb{R}^2$ fixed),

(ii) $S = \mathbb{C} \ni \rho e^{i\theta} \mapsto \rho e^{i(\theta + r)}$

are group actions and describe their orbits geometrically. What do stabilizers look like?

Question 3 Let $H \leq G$ with $[G : H] = n$. Show that G has a normal subgroup of index at most $n!$.

Question 4 Find the center of $Gl(2, \mathbb{R})$, the dihedral groups, and $SO(3, \mathbb{R})$.

Question 5 Show that $G/Z(G)$ cyclic $\Rightarrow G$ is abelian. Show that if p is prime, then all groups of order p^2 are abelian.

Question 6 *Research problem – answers must be less than 1 page.* What are (i) a Lie group, (ii) a Lie algebra and (iii) a Dynkin diagram?