250A Homework 10

Ungraded, solutions to appear soon...

Question 1 (*Internal direct sum/product*) Show that if a ring $R = J_1 + \cdots + J_n$ and the ideals $J_i \cong R_i$ for rings R_i and $J_i \cap \sum_{j \neq i} J_j = \{0\}$, then $R \cong \otimes_{i=1}^{n} R_i$.

Question 2 Look up Zorn's lemma on partially ordered sets (*e.g.* Dummit and Foote Appendix I). Let R be a ring with 1 and at least one proper ideal (an ideal other than R or $\{0\}$). Now prove that R has a maximal proper ideal (*i.e.* not contained in any other proper ideal)[1].

Question 3 *Frobenius homomorphism* Let R be a commutative ring with identity and prime characteristic p. Show that the map

$$\varphi : R \longrightarrow R, \, , \quad r \longmapsto r^p \, ,$$

is a ring homomorphism.

Question 4 Find all ideals in $\mathbb{Z}$.

[1] Hint: study "chains" in $\{J | I \subseteq J \subseteq R\}$ for some proper ideal I.