# 250A Homework 2

Due Monday October 18

**Question 1** Let $G$ be a group and suppose $x^2 = 1 \ \forall x \in G$. Prove that $G$ is abelian.

**Question 2** Let $G = \{x_1, \ldots, x_n\}$ be a finite abelian group. Prove

$$(x_1 \cdots x_n)^2 = 1.$$

**Question 3** Find all groups of order 7 or less.

**Question 4** Let $\mathbb{R}^+$ and $\mathbb{R}^*$ be the group of real numbers under addition and non-zero real numbers under multiplication, respectively. Show that these groups are not isomorphic.

**Question 5** Show that the $n$th roots of unity of $\{z \in \mathbb{C} | z^n = 1, n \in \mathbb{N}\}$ form a group under multiplication but not addition.

**Question 6** Let $x \in G$ a group. Show that $|x| = n < \infty$ implies that $1, x, x^2, \ldots, x^{n-1}$ are distinct.

**Question 7** Suppose $\mathcal{N}$ is a nilpotent operator on a vector space $V$. In this case $V$ is a direct sum of cyclic subspaces. Show that the number of summands is $\dim \ker \mathcal{N}$.