250A Homework 3

Due Monday October 25

Question 1 Definition: $\operatorname{tor}(G) = \{g \in G | g^n = 1 \text{for some } n \in \mathbb{N}\}$. In addition a group G is called torsion-free if $\operatorname{tor}(G) = \{1\}$.

Let Γ be a group which contains a torsion-free subgroup Γ_0 of index $n < \infty$. Show that Γ does not contain any element whose order is finite and strictly larger than n.

Question 2 Show that $\operatorname{tor}(G) \leq G$ if G is abelian.

Question 3 Show that the order of a permutation is the least common multiple of the lengths of its disjoint cycles.

Question 4 Find all normal subgroups of S_4.

Question 5 Show that $K \leq H \leq G$ and $K \trianglelefteq G \Rightarrow K \trianglelefteq H$. Now, a permutation is called even if it can be written as an even number of transpositions (e.g. (132)=(12)(13) is even). The alternating group A_n is the subgroup of the symmetric group S_n consisting of only even permutations. Study the group A_4 to decide whether $K \trianglelefteq H \trianglelefteq G \Rightarrow K \trianglelefteq G$.

Question 6 Show that $Sl(n, \mathbb{R}) \trianglelefteq Gl(n, \mathbb{R})$.