250A Homework 6

Due Monday November 15

Question 1 Let G act on the set S. Show that if $s, t \in S$ belong to the same orbit, their stabilizers are conjugate subgroups of G.

Question 2 Let x and y be conjugate elements in a finite group G. Show that the number of distinct $g \in G$ with $g^{-1}xg = y$ equals the order of the normalizer of x.

Question 3 Which finite groups have precisely one or two conjugacy classes?

Question 4 Fix $x \in G$ a group. Show that $G \ni g \mapsto xgx^{-1} \in G$ is an automorphism of G and in turn demonstrate that $G/Z(G)$ is isomorphic to a subgroup of $\mathrm{Aut}(G)$.