250A Homework 7

Due Monday November 22

Let p be prime throughout.

Question 1 Let $H \trianglelefteq G$ such that $p \nmid [G : H]$. Show that every Sylow-p-subgroup of G is contained in H.

Question 2 Let $K \trianglelefteq G$ be a p-subgroup. Show K is contained in every Sylow-p-subgroup of G.

Question 3 Decide upon the validity of the following statements. Give reasons (*i.e.* proofs or counterexamples) for your answers.

(i) If $H \leq G$ and S is a Sylow-p-subgroup of G, then $S \cap H$ is a Sylow-p-subgroup of H.

(ii) If S and S' are Sylow-p-subgroups of G and G', respectively, then $S \times S'$ is a Sylow-p-subgroup of $G \times G'$.

(iii) If G has a Sylow-p-subgroup of order p^n, then all subgroups of order p^{n-1} are conjugate.

Question 4 If $H \leq G$ and P, P' are distinct Sylow-p-subgroups of H, show P and P' cannot be subgroups of the same Sylow-p-subgroup of G.

Question 5 Recall that a simple group G has no normal subgroups save for itself and 1_G. How many simple groups are there of the following orders: 18, 56, 70, 225 and 700? Give reasons for your answer in each case.

Question 6 Suppose p and q are distinct primes. Show there are no simple groups of order $p^2 q$.