250A Homework 8

Due Monday November 29

Question 1 Comment on the (historical) significance of the solvability of S_5.

Question 2 Compute the derived series for dihedral groups D_n and their quotients. Are these groups solvable? Of what length?

Question 3 As in question 2 but upper triangular invertible $n \times n$ matrices.

Question 4 Find an example of both a finite and infinite group with a quotient group not isomorphic to any subgroup.

Question 5 Let p be prime and F_p the field with p elements[1].

(i) Let $N \leq Gl(3, F_p)$ be the group of 3×3 F_p-valued upper triangular matrices with 1's on the diagonal. Show that G is non-abelian of order p^3.

(ii) Compute the order of $Gl(3, F_2)$. Identify the Sylow-p-subgroups and decide if any of them are normal.

[1]Take integers mod p with multiplication and addition defined in the usual way.