250A Homework 9

Due Monday December 6

Question 1 (DF 7.1.15) Rings R where $a^2 = a \; \forall \; a \in R$ are called Boolean. Prove that Boolean rings are abelian and $a + a = 0 \; \forall \; a \in R$.

Question 2 Let R be a commutative ring and call $x \in R$ nilpotent if $x^n = 0$ for some $n \in \mathbb{N}$. Prove that the set of nilpotent elements is an ideal I (i.e. an additive subgroup closed under left and right multiplication). Show that the quotient R/I has no non-trivial nilpotent elements. Give an example of a ring in which the set of nilpotent elements is not an ideal.

Question 3 Let R be a ring and $f, g : \mathbb{Q} \to R$ be ring homomorphisms coinciding on the integers, i.e. $f|_{\mathbb{Z}} = g|_{\mathbb{Z}}$. Show that $f = g$.

Question 4 Give examples of rings R where (i) R has a left ideal that is not a right ideal (ii) R has zero divisors but R/I does not for some ideal I.