Change of Orthonormal Basis Worksheet

In the change of basis worksheet, we saw that a pair of bases $(v_1, \ldots, v_n)$ and $(v'_1, \ldots, v'_n)$ for a vector space V could be related by a invertible change of basis matrix P defined by

$$(v'_1, \ldots, v'_n) = (v_1, \ldots, v_n)P \,.$$

The columns of P were calculated by computing the component of the vectors $(v'_1, \ldots, v'_n)$ in the basis $(v_1, \ldots, v_n)$. Also if M is the matrix of $f \in \mathrm{end}(V)$ in the basis $(v_1, \ldots, v_n)$, its matrix M' in the basis $(v'_1, \ldots, v'_n)$ is given by

$$M' = P^{-1}MP \,.$$

If you are lucky enough to have orthonormal bases, life is easier..... For example, consider $\mathbb{C}^3$ with standard inner product $\langle (z_1, z_2, z_3), (w_1, w_2, w_3) \rangle = \sum_{i=1}^{3} \bar{w}_i z_i$. An orthonormal basis is $f_1 = (1, i, 0)/\sqrt{2}, f_2 = (1, -i, 0)/\sqrt{2}, f_3 = (0, 0, 1)$. To compute the components of a vector v in this basis, you only have to compute $\langle v, f_1 \rangle$, $\langle v, f_2 \rangle$, $\langle v, f_3 \rangle$. *Calculate the components of the vector $v = (1, 1, 1)$ in the basis (f_1, f_2, f_3) using the inner product.*

Now let $e_1 = (1, 0, 0), e_2 = (0, 1, 0), e_3 = (0, 0, 1)$ be the canonical (orthonormal) basis for $\mathbb{C}^3$. *Calculate the change of basis matrix P from the basis (e_1, e_2, e_3) to (f_1, f_2, f_3).* Remember that $P^\dagger$ is obtained from P by taking the transpose and complex conjugate. *Compute $P^\dagger$ and $P^\dagger P$. What can you say about $P^\dagger$? EXPLAIN YOUR OBSERVATION!*