

Quiz 4. Problem: Chapter 6 PWE.2.

To check $\{T(v_1), \dots, T(v_n)\}$ is linearly independent, let

$$c_1 T(v_1) + c_2 T(v_2) + \dots + c_n T(v_n) = 0.$$

Since T is linear.

$$T(c_1 v_1 + c_2 v_2 + \dots + c_n v_n) = 0.$$

Since T is injective, $\ker T = \{0\}$, then

$$c_1 v_1 + c_2 v_2 + \dots + c_n v_n = 0.$$

Since $\{v_1, v_2, \dots, v_n\}$ is linearly independent,

$$c_1 = 0 \quad c_2 = 0 \quad \dots \quad c_n = 0.$$

In Summary, the only solution to $c_1 T(v_1) + c_2 T(v_2) + \dots + c_n T(v_n) = 0$ is $c_1 = 0 \quad c_2 = 0 \quad \dots \quad c_n = 0$, so $\{T(v_1), T(v_2), \dots, T(v_n)\}$ is linearly independent.