Math 67A Quiz 3 Solution

Joe Grimm

February 14, 2012

Problem Let V be a vector space over $\mathbb{F}$ and suppose that $\{v_1, v_2, .., v_n\}$ is linearly independent. Prove or disprove: $\{v_1 - v_2, v_2 - v_3, ..., v_{n-1} - v_n, v_n\}$ is also linearly independent.

Solution The set $\{v_1 - v_2, v_2 - v_3, ..., v_{n-1} - v_n, v_n\}$ is linearly independent if the only solution to

$$a_1(v_1 - v_2) + a_2(v_2 - v_3) + ... + a_{n-1}(v_{n-1} - v_n) + a_n v_n = 0$$

is $a_1 = a_2 = ... = a_n = 0$. I re-group the coefficients to write a linear combination of the original v_i

$$a_1 v_1 + (a_2 - a_1)v_2 + ... + (a_n - a_{n-1})v_n = 0.$$

Since $\{v_1, v_2, .., v_n\}$ is linearly independent, the only solution to the above equation is $a_1 = a_2 - a_1 = ... = a_n - a_{n-1} = 0$. Since $a_1 = 0$ and $a_2 - a_1 = 0$ we have that $a_2 = 0$ and so on for all a_i; thus, the only solution to

$$a_1(v_1 - v_2) + a_2(v_2 - v_3) + ... + a_{n-1}(v_{n-1} - v_n) + a_n v_n = 0$$

is the trivial solution and the set $\{v_1 - v_2, v_2 - v_3, ..., v_{n-1} - v_n, v_n\}$ is linearly independent.