Math 67 Quiz 2 Solution

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1 Show that $W = \{(x, y, z) : x + y + z = 0\}$ is a subspace of $\mathbb{R}^3$.

solution A subspace of a vector space is a subset of a vector space that is closed under scalar multiplication and vector addition. To check closure under scalar multiplication choose some $\lambda \in \mathbb{R}$ and $(x_1, y_1, z_1) \in W$, then

$$\lambda(x_1, y_1, z_1) = (\lambda x_1, \lambda y_1, \lambda z_1),$$

and

$$\begin{aligned} \lambda x_1 + \lambda y_1 + \lambda z_1 =& \lambda(x_1 + y_1 + z_1) \\ =& \lambda \cdot 0 \\ =& 0. \end{aligned}$$

Thus, $\lambda(x_1, y_1, z_1) \in W$ so W is closed under scalar multiplication.

To check closure under vector addition consider $(x_1, y_1, z_1), (x_2, y_2, z_2) \in W$ and take the sum

$$(x_1, y_1, z_1) + (x_2, y_2, z_2) = (x_1 + x_2, y_1 + y_2, z_1 + z_2).$$

The following computation shows that this sum is in W

$$\begin{aligned} x_1 + x_2 + y_1 + y_2 + z_1 + z_2 =& (x_1 + y_1 + z_1) + (x_2 + y_2 + z_2) \\ =& 0 + 0 \\ =& 0. \end{aligned}$$

2 Show that $U = \{(x, y, z) : x + y + z = 1\}$ is not a subspace of $\mathbb{R}^3$.

solution The set U is not a subspace because it is not closed under scalar multiplication. Take $\vec{x} = (1, 0, 0) \in U$ then $2\vec{x} = (2, 0, 0) \notin U$. In fact, the sum of any two elements of U will be a vector not in U, as will any non-unit multiple of an element of U.