

## Section 5.4, number 20

(a) The average fitness function is derived by substituting  $q = 1 - p$  and simplifying,

$$\bar{w}(p) = p^2(w_{11} - 2w_{12} + w_{22}) + 2p(w_{12} - w_{22}) + w_{22}. \quad (1)$$

The plots for the three cases are on the next page.

(b) The derivative:

$$\frac{d\bar{w}}{dp} = 2p(w_{11} - 2w_{12} + w_{22}) + 2(w_{12} - w_{22}) \quad (2)$$

(c) The critical number for  $\bar{w}$ :

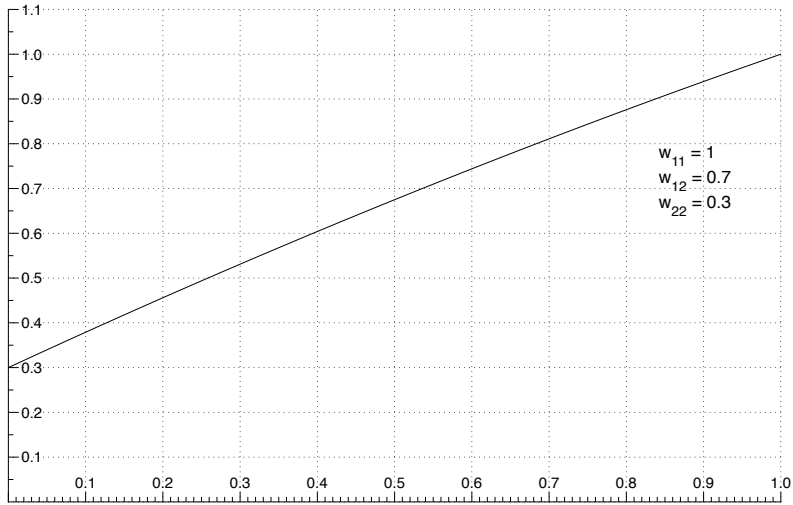
$$p = \frac{w_{22} - w_{12}}{w_{11} - 2w_{12} + w_{22}}. \quad (3)$$

The domain for  $p$  is  $0 \leq p \leq 1$ . For case (i), the critical number is 4.  $\bar{w}(4) = 1.9$ ,  $\bar{w}(0) = 0.3$ ,  $\bar{w}(1) = 1$ . The global maximum is at  $p = 1$  because the critical number is outside the domain.

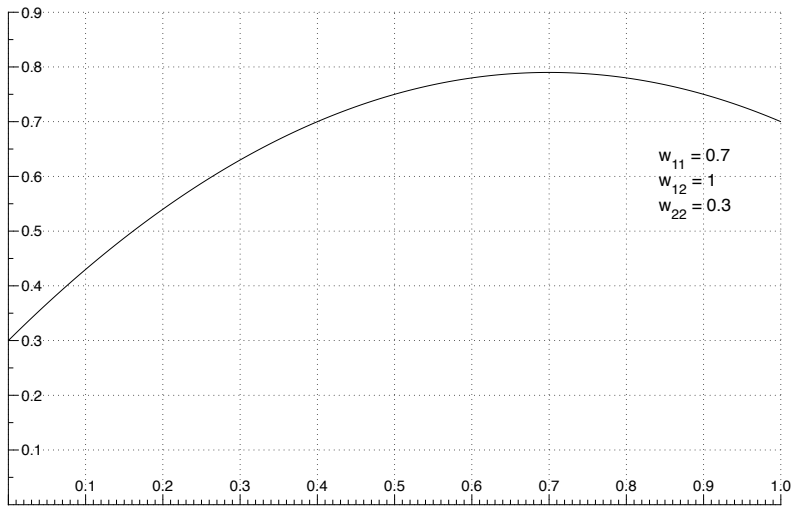
For case (ii), the critical number is  $7/10$ .  $\bar{w}(7/10) = 0.79$ ,  $\bar{w}(0) = 0.3$ ,  $\bar{w}(1) = 0.7$ . The global maximum is at  $p = 0.7$ .

For case (iii), the critical number is  $4/11 \approx 0.363636$ .  $\bar{w}(4/11) = 61/110 \approx 0.554545$ ,  $\bar{w}(0) = 0.7$ ,  $\bar{w}(1) = 1$ . The global maximum is at  $p = 1$ .

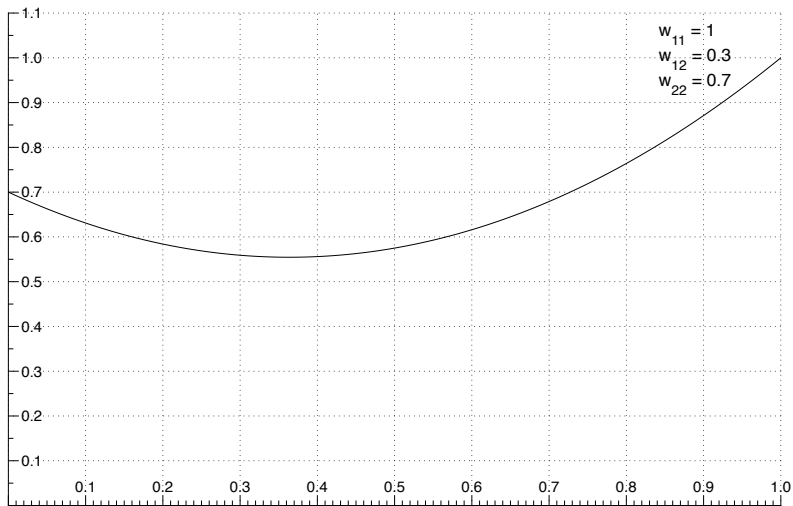
(d) The equilibrium frequency is where the derivative is zero. Case (i):  $p = 4$ , case (ii),  $p = 0.7$ , and for case (iii),  $p = 4/11$ .



(i) Directional selection



(ii) Overdominance



(iii) Underdominance