

MAT 17A Section A02
Extra Credit Worksheet
October 22, 2009

Name: _____ SID: _____

Instructions:

1. There is no partial credit. Each problem is worth one point.
2. The worksheet is due at the end of the discussion section.
3. Your TA will help you set up the problem, but will not tell you the answer.
4. Read each problem carefully and show all work as neatly as possible
5. No credit will be given for correct answers without supporting work.
6. Please circle your final answers. Make sure that you label your graphs as instructed.

- 1.** Use the bisection algorithm to find a value for x that satisfies

$$\exp(-x^2) - \frac{1}{5} \sin(\pi x) = 0.$$

Fill in the table below (the interval is $a = 1.5$ and $b = 2.5$). Keep at least four decimal places.

a	$(a + b)/2$	b	$f(a)$	$f((a + b)/2)$	$f(b)$
1.5	2.0	2.5	0.3053992	0.0183156	-0.1980695
2.0	2.25	2.5	0.0183156	-0.135092	-0.1980695
2.0	2.125	2.25	0.0183156	-0.0655999	-0.135092

2. Supposed you want to ensure that the error in the bisection method is less than 10^{-6} . Given an interval $[a, b] = [2, 6]$, calculate how many steps of the bisection method would be needed.

$$\mathbf{Hint: Error} \leq |b - a| \left(\frac{1}{2}\right)^n$$

$$\begin{aligned} 10^{-6} &\leq |6 - 2| \left(\frac{1}{2}\right)^n \\ \frac{10^{-6}}{4} &= \left(\frac{1}{2}\right)^n \\ \ln\left(\frac{10^{-6}}{4}\right) &= n \ln(0.5) \\ n &= \frac{\ln(10^{-6}/4)}{\ln(0.5)} \approx 21.9316 \approx 22 \end{aligned}$$

Answer: 22 steps

3. Evaluate the following limit using the Squeeze theorem.

$$\lim_{x \rightarrow 0} \left(x^4 \cos\left(\frac{2}{x}\right) \right)$$

$$-1 \leq \cos\left(\frac{2}{x}\right) \leq 1$$

$$-x^4 \leq x^4 \cos\left(\frac{2}{x}\right) \leq x^4$$

$$\lim_{x \rightarrow 0} -x^4 = 0 \text{ and } \lim_{x \rightarrow 0} x^4 = 0$$

Therefore $\lim_{x \rightarrow 0} \left(x^4 \cos\left(\frac{2}{x}\right) \right) = 0$ by the Squeeze theorem.

4. Find the equation of the secant line to the curve $f(x)$ using the x -values: $x = 1/2$ and $x = 1$. **Hint:** Find the equation of the line connecting $(x_1, f(x_1))$ and $(x_2, f(x_2))$. Type in $\tanh(0.5)$ in your calculator to get the value at x_1 .

$$f(x) = \tanh(x)$$

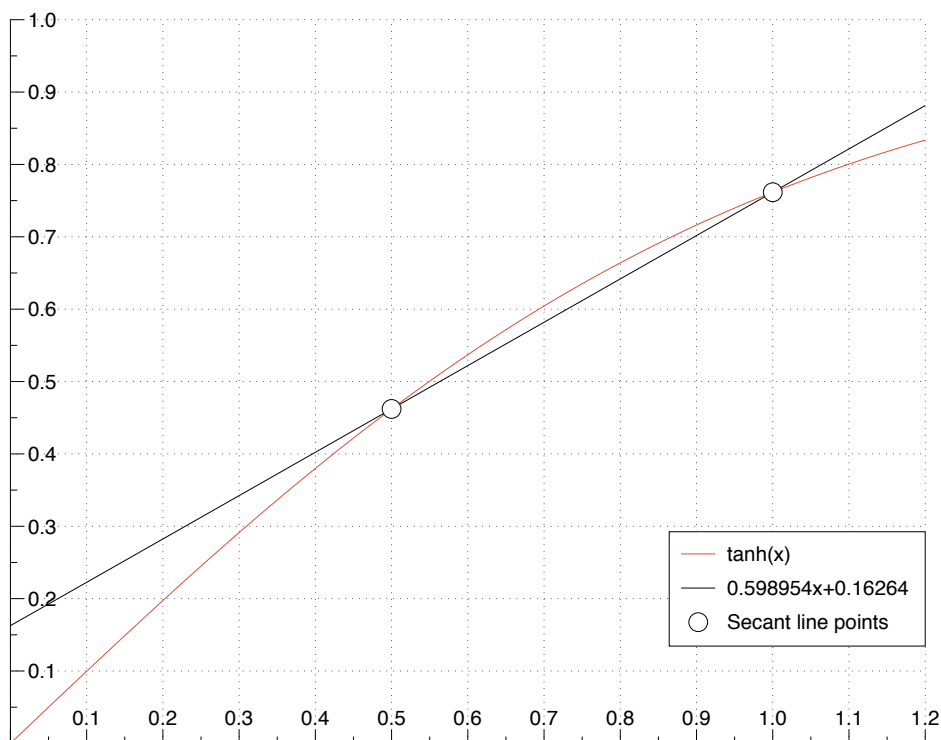
$$\tanh(0.5) = 0.462117 \text{ and } \tanh(1) = 0.761594$$

Therefore, our two points on the curve are $(0.5, 0.462117)$ and $(1, 0.761594)$.

$$m = \frac{\Delta y}{\Delta x} = \frac{0.761594 - 0.462117}{1 - 0.5} = \frac{0.299477}{0.5} = 0.598954,$$

$$y - 0.761594 = 0.598954(x - 1)$$

$$y = 0.598954x + 0.16264$$



5. A strain of bacteria reproduces asexually every 30 minutes. That is, every 30 minutes, each bacteria cell splits into two cells. If initially there are 40 bacteria, how long will it take to until there are 1280 bacteria?

$$N_t = N_0 R^t, N_0 = 40$$

Because the bacteria are doubling, $R = 2$.

You could also use the recursion $N_{t+1} = 2N_t$ with $N_0 = 40$.

$$1280 = 40 \times 2^t$$

$$32 = 2^t$$

$$\ln(32) = t \ln(2)$$

$$t = 5 \text{ time steps} = 5 \times 30 = 150 \text{ minutes} = 2 \text{ hours and } 30 \text{ minutes.}$$