Exam 2, Version 2

MAT 17A: sections A01-A02

Instructor: Wanda Strychalski

November 20, 2009

Name: ______________________________

ID Number: ____________________________

Circle your section number:

Section	Time	TA
A01	5:10	Han Wang
A02	6:10	Euna Chong

Directions:

- Do not begin until instructed to do so.
- You may use pencils, pens, erasers, and calculators.
- Put away all books, notes, cell phones, or electronic devices.
- Turn off your cell phone.
- Show all work, clearly and in order to receive full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- **Leave your answers in exact form unless otherwise instructed** (i.e. $\sqrt{2}$ not 1.41).
- Circle or otherwise indicate your final answers.
- You must present your student ID when turning in your exam.
- This test has 6 problems and 8 pages.
- Good luck!

I pledge that I have not received unauthorized assistance on this exam.

Signature: ______________________________

Name: ____________________

1. (*8 points*) Differentiate the following functions with respect to the independent variable.

(a) (*2 points*) $g(x) = \dfrac{x^3+1}{1-2x}$

$$g'(x) = \frac{(1-2x)(3x^2)-(x^3+1)(-2)}{(1-2x)^2}$$

$$g'(x) = \frac{3x^2-6x^3+2x^3+2}{(1-2x)^2}$$

$$g'(x) = \frac{-4x^3+3x^2+2}{(1-2x)^2}$$

(b) (*3 points*) $h(t) = \sqrt[3]{t}\Big(\ln(6t)+4\Big)$

$$h'(t) = \sqrt[3]{t}\frac{6}{6t} + \frac{1}{3}t^{-2/3}(\ln(6t)+4)$$

$$h'(t) = \frac{\sqrt[3]{t}}{t} + \frac{\ln(6t)+4}{3t^{2/3}}$$

$$h'(t) = \frac{1}{t^{2/3}} + \frac{\ln(6t)+4}{3t^{2/3}}$$

(c) (*3 points*) $f(x) = \sin(\sin x)$

$$f'(x) = \cos(\sin(x))(\cos(x))$$

$$f'(x) = \cos(x)\cos(\sin(x))$$

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2. (*8 points*) Given the function $f(x) = \frac{x^4}{2} - \frac{x^3}{3} - \frac{x^2}{2} - 1$,

(**a**) (*4 points*) Find the critical numbers of the function.

$$f'(x) = 2x^3 - x^2 - x$$

$$2x^3 - x^2 - x = 0$$

$$x(2x^2 - x - 1) = 0$$

$$x = 0 \text{ or } 2x^2 - x - 1 = 0$$

$$x = 0 \text{ or } (2x+1)(x-1) = 0$$

$$\text{Ctitical numbers: } -\frac{1}{2}, 0, 1$$

(**b**) (*4 points*) Find the absolute maximum and minimum values of f on the interval $[-2, 2]$. Round to two decimal places.

x	-2	$-1/2$	0	1	2
$f(x)$	7.667	-1.0521	-1	-1.333	2.333

The absolute maximum is at $(2, 7.667)$.

The absolute minimum is at $(1, -1.333)$.

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3. (*6 points*) Find an equation of the tangent line to the curve

$$x^{2/3} + y^{2/3} = 4$$

at the point $\left(3\sqrt{3}, 1\right)$. The line should be given in $y = mx + b$ form. HINT: $\sqrt[3]{3\sqrt{3}} = \sqrt{3}$.

$$\begin{aligned} \frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}\frac{dy}{dx} &= 0 \\ \frac{2}{3}y^{-1/3}\frac{dy}{dx} &= -\frac{2}{3}x^{-1/3} \\ \frac{dy}{dx} &= -\frac{x^{-1/3}}{y^{-1/3}} \\ \frac{dy}{dx} &= -\sqrt[3]{\frac{y}{x}} \end{aligned}$$

The slope of the tangent line at $\left(3\sqrt{3}, 1\right)$ is $\left.\frac{dy}{dx}\right|_{(3\sqrt{3},1)} = -\sqrt[3]{\frac{1}{3\sqrt{3}}} = -\frac{1}{\sqrt[3]{3\sqrt{3}}} = -\frac{1}{\sqrt{3}}$.

$$\begin{aligned} y - 1 &= -\frac{1}{\sqrt{3}}\left(x - 3\sqrt{3}\right) \\ y &= 1 - \frac{x}{\sqrt{3}} + 3 \\ y &= -\frac{x}{\sqrt{3}} + 4 \end{aligned}$$

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4. (*10 points*) Verify that the function $f(x) = 2x^3 + x - 4$ satisfies the hypotheses of the Mean Value Theorem on the interval $[0, 1]$. Then find all numbers c that satisfy the conclusion of the Mean Value Theorem.

Because $f(x)$ is a polynomial, it is continuous on $[0, 1]$ and differentiable on $(0, 1)$. Therefore $f(x)$ satisfies the hypotheses of the Mean Value Theorem, and

$$f'(c) = \frac{f(1) - f(0)}{1 - 0}$$

$$f'(x) = 6x^2 + 1$$

$$f(1) = -1$$

$$f(0) = -4$$

$$6c^2 + 1 = \frac{-1 + 4}{1}$$

$$6c^2 = 2$$

$$c = \pm\sqrt{\frac{1}{3}}$$

$c = \dfrac{1}{\sqrt{3}}$ because the negative value of c is not in the domain.

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5. (*8 points*) Use logarithmic differentiation to find $\frac{dy}{dx}$. Your answer does not need to be simplified.

$$y = \frac{\left(x^2+1\right)^4 3^x}{\sqrt[3]{x}\cos x}$$

$$\ln(y) = 4\ln(x^2+1) + x\ln 3 - \left[\frac{1}{3}\ln x + \ln(\cos x)\right]$$

$$\ln(y) = 4\ln(x^2+1) + x\ln 3 - \frac{1}{3}\ln x - \ln(\cos x)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{4\cdot 2x}{x^2+1} + \ln 3 - \frac{1}{3x} - \frac{-\sin x}{\cos x}$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{8x}{x^2+1} + \ln 3 - \frac{1}{3x} + \tan x$$

$$\frac{dy}{dx} = y\left(\frac{8x}{x^2+1} + \ln 3 - \frac{1}{3x} + \tan x\right)$$

$$\frac{dy}{dx} = \frac{\left(x^2+1\right)^4 3^x}{\sqrt[3]{x}\cos x}\left(\frac{8x}{x^2+1} + \ln 3 - \frac{1}{3x} + \tan x\right)$$

6. (*10 points*) Consider a snowball with surface area $S(r) = 4\pi r^2$.

(**a**) (*6 points*) If a snowball melts so that its surface area decreases at a rate of 0.6 cm^2/min, find the rate at which the diameter decreases when the diameter is 5 cm. Round your answer to three decimal places.

If diameter D is $D = 2r$,

$$\frac{dD}{dt} = 2\frac{dr}{dt} \text{ or } \frac{1}{2}\frac{dD}{dt} = \frac{dr}{dt}$$

$$S(D) = 4\pi\left(\frac{D}{2}\right)^2$$

$$S(D) = \pi D^2$$

$$\frac{dS}{dt} = 2\pi D\frac{dD}{dt}$$

$$-0.6 = 2\pi(5)\frac{dD}{dt}$$

$$\frac{dD}{dt} = -\frac{0.6}{10\pi} \approx -0.0191$$

(**b**) (*4 points*) If the radius of the snowball is measured to be $r = 2 \pm 0.1$, what is the interval of the accuracy of the surface area $[S - \Delta S, S + \Delta S]$? Round your answer to three decimal places.

$$\Delta S = S'(r)\Delta r = 8\pi r\Delta r$$

$$\Delta S = S'(r)\Delta r = 8\pi(2)(0.1) = 1.6\pi$$

$$S(3) = 4\pi \times 2^2 = 16\pi$$

$$[50.266 - 5.027, 50.266 + 5.027]$$

$$[45.239, 55.292]$$

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Bonus Question (2 Extra Credit Points): You are riding your bicycle and approach a traffic circle. Which of the following actions should you take (circle 1 or 2)?

1. ◯ Slow down and yield to the traffic already in the circle.
2. Enter the traffic circle and assume that other people will get out of your way.