Information Theoretic Approaches to Spiking Neural Models

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Introduction

Recent studies focusing on the computational capabilities of neurobiological networks have suggested that information may often be encoded in a more precise way than can be represented by typical spike-rate based artificial neural networks. Temporal resolutions of neural codes commonly exist on a millisecond time scale, which highlights the significance of the precision of individual spikes in the coding process.

The work presented here is an early step towards the goal of characterizing, in a quantitative fashion, the range of behaviors possible in spiking networks of various connectivity. Specifically, for small networks of six neurons, I have systematically examined and compared **entropies** in the behavior of networks with a range of connectivity patterns and noted trends in the types of spike patterns with which these entropies are correlated. Additionally, I have studied **mutual information** between spike trains of individual neurons in small networks and made conclusions regarding how structural connectivity may be reflected within such behavioral measurements.

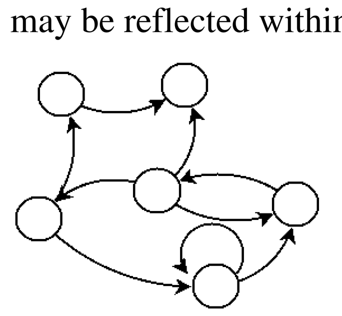


Figure 1. Diagram of a typical six neuron network. Circles represent neurons and arrows represent synaptic connections.

Methods

Integrate-and-fire model neurons provide one of the simplest descriptions of how a cell may receive, integrate, and leak synaptic input currents arriving from neighboring neurons.

$$\tau \frac{dV}{dt} = V_{\text{rest}} - V + R_m I_e(t) \qquad (1)$$

(1) Equation describing dynamics of single neuron with membrane voltage V.

$$V(t_+) = V_{\text{reset}} \quad \textit{when} \quad V(t_-) = V_{\text{thresh}} \qquad (2)$$

(2) Spiking and reseting upon reaching threshold.

$$I_e(t) = I_{\text{exc}} + I_{\text{inh}} + I_{\text{app}} \qquad (3)$$

(3) Inputs. Note only excitatory input is used here.

Exploring and comparing entropy of networks

100 trials of size six networks with random connectivity between 25% and 75% (i.e. the probability of an edge existing between any two nodes) were run, each for 3000 time steps with the first 50 time steps disregarded as transient. Maximum and minimum entropy was calculated among the trials (treating a block of neuron outputs at a single time step as a symbol) and the associated networks and spike patterns were noted and analyzed.

Exploring shifted mutual information between neurons in a network

Pairwise mutual information was calculated on outputs of all pairs of neurons in a size four and five network, for temporal shifts of zero to five time steps both directions.

Results

Results presented here highlight the exploration of entropy in the behavior of networks of six neurons as well as the investigation of mutual information calculated between the neuron outputs in two specific networks of size four and five.

Network Entropy

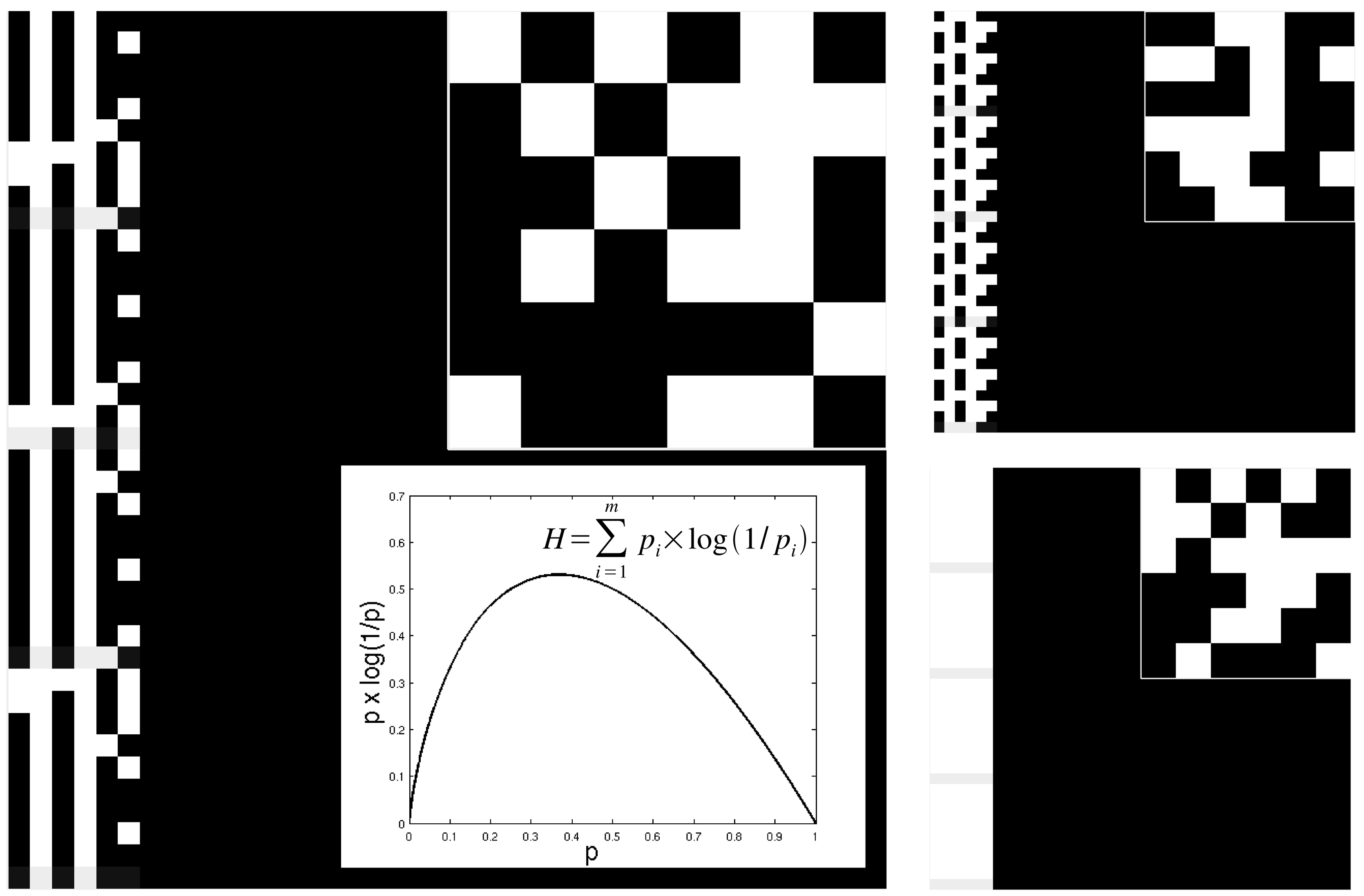


Figure 2. Screenshots of program GUI. Leftmost block sequence represents the spike train: a record of spikes over time. Discretized time is on vertical axis and neurons are across the horizontal. White represents spiking. Top right square binary matrix represents connectivity. White indicates there is a synapse from the neuron identified by the column to the one identified by the row. ***Left***: Network and output corresponding to largest observed entropy; produced through diverse patterns of firing (attributable to right most neuron). *Inset:* Graph of p*log(p), used in calculation of entropy, H. ***Top right***: Large entropy produced through cyclic (specifically 3-cycle) behavior. ***Bottom right***: Small entropy resultant from homogeneous output. **Note:** initial conditions for all networks is every node spiking.

The first two cases above are representative of the two general ways in which networks obtained larger than average entropy:

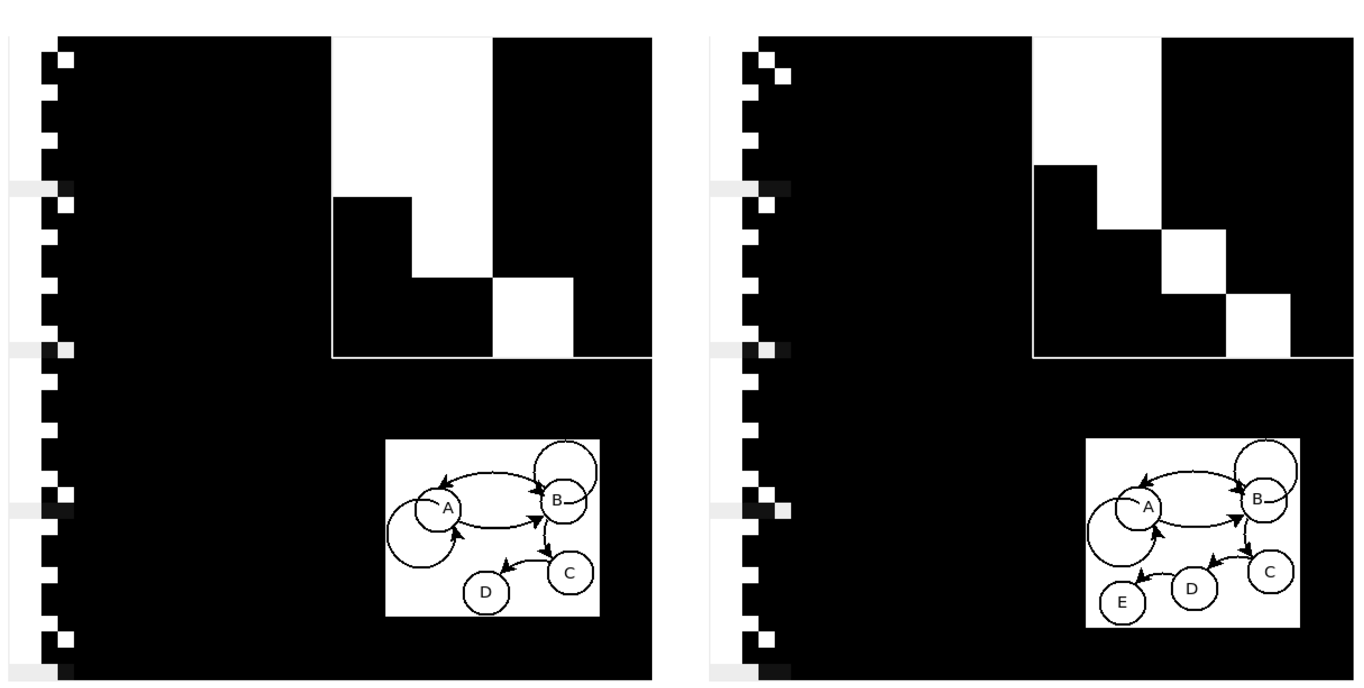
(1) Produce spike trains containing a relatively large number of different symbols out of the 2^6=64 possible in the alphabet (here "alphabet" refers to the set of permutations of possible network activity at one time). Probability distribution shapes in this case were non-uniform.

(2) Produce symbols which appear almost with uniform probability. This method was concurrent with the existence of few symbols, almost always appearing in the form of a small cycle (3 or 4 unique patterns in series typically) over time.

Clearly, **(1)** maximizes the number of indicies in the summation, *m*; while the **(2)** maximizes what is inside the sum. Interestingly, it appeared, at least in these trials, that six node networks of this variety were unable to simultaneously maximize diversity as well as homogeneity in the frequency distributions of the symbols. The reasons for this have not been uncovered.

Shifted Mutual Information Between Neurons

$$I(X;Y) = H(X) - H(X/Y)$$



Node1	Node2	Time shift	Mutual information
C	D	-1	0.197
C	D	2	0.197
C	D	otherwise	0.070
D	E	-1	0.126
D	E	otherwise	0.006
C	E	-2	0.060
C	E	1	0.060
C	E	otherwise	0.022
All other combinations		All shifts	0.000

Figure 3 and Table 1. Demonstration that mutual information, I, calculated pairwise between time shifted neuron outputs in a spiking network may uncover the causal structure. Note values involving only neurons A, B, C and D are same for both networks.

Conclusions

It is known that **Shannon Entropy** will be maximum when every symbol out of an alphabet is present in a sample distribution with equal probability. As explored here, I have found that both in persistent and transient (not shown) trials, the behavior of networks which maximize entropy relative to other networks tested fall into *two main categories*: those which contain many patterns but in unequal probabilities or those which have equal probabilities but few patterns. This result is compelling food for future research. One particularly pressing question is whether the trend seen of a failure to use both strategies for maximizing entropy is indicative of a fundamental limitation on these small networks or whether there are in fact some specific networks which are able to close the gap on entropy maximization.

Just as the experiments exploring **mutual information** in the designed four node network gave compelling evidence that special analysis of the spike trains coming out of the network may uncover structure in the connectivity of the network, experiments with the five node network are suggestive of a further fact (compare the two highlighted values in Table 1). Namely that the **data processing inequality** which is universally true of **Markov chains** may also be true in some form for some types of spiking neural networks. That is to say if there is a one way chain of causal effect A->B->C (where A, B and C are neurons), the information in the behavior of A is more informative of that of B (and vice-versa) than it is of C. The idea is intuitive but more work is necessary to show conclusively that it holds in all cases in spiking neural networks.

Next Steps

- **2D Block entropy –** In order to study patterns that exist in time, treat NxK blocks (where K is a number of time steps) as symbols in an alphabet.
- **Response to input –** Study how networks differentially respond to various kinds of input. Study how a single network responds to the same input while in different states of activity.
- **Mapping structural to behavioral patterns –** Find consistent mappings between structural patterns of connectivity in networks and the resultant behavior.

For further information

Please contact **watson@math.ucdavis.edu** with questions or comments. More information and background (including Python code) on this project can be obtained at **http://www.math.ucdavis.edu/watson/spiking/info**