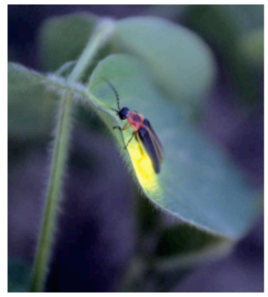
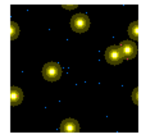
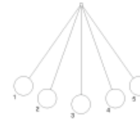
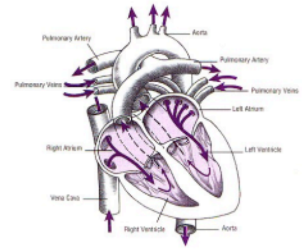
# Synchronization of Biological Oscilators









Handout for Oakland / East Bay Math Circle
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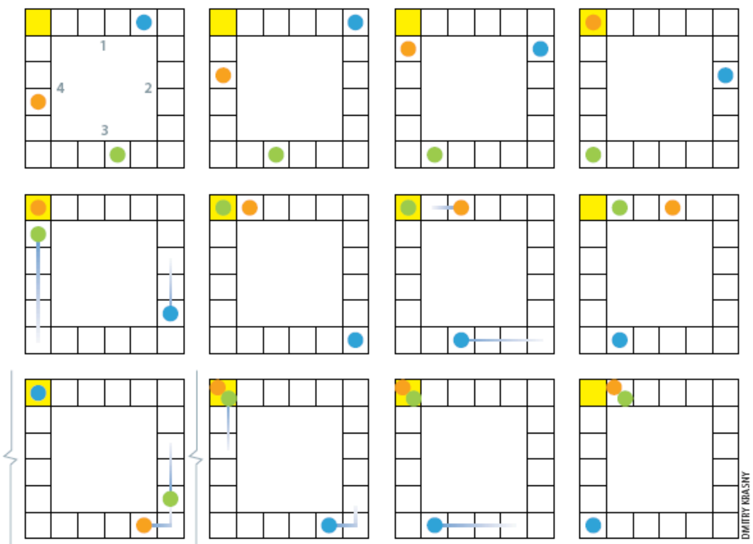
Contact Watson at watson@math.ucdavis.edu
see below for website with links and videos

From: The Synchronicity of Firefly Flashing by Ian Stewart
Mathematical Recreations. 104. Scientific American March 1999.




GAME OF FLASH
*approximates firefly behavior. In this sequence of opening stages, which go from left to right and top to bottom, three fireflies* (colored dots) *move clockwise, approaching the "flash" square* (yellow). *When a firefly reaches that square, the insect emits a flash of light, which then makes all other fireflies skip closer* (blue lines) *to flashing themselves. Several stages of the game have been omitted* (gray lines).

*Imagine a tree thirty-five to forty feet high ... apparently with a firefly on every leaf and all the fireflies flashing in perfect unison at the rate of about three times in two seconds, the tree being in complete darkness between flashes....*

*Imagine a tenth of a mile of river front with an unbroken line of mangrove trees with fireflies on every leaf flashing in synchronism, the insects on the trees at the ends of the line acting in perfect unison with those between. Then, if one's imagination is sufficiently vivid, he may form some conception of this amazing spectacle.*

From: Synchronization of pulse-coupled biological oscillators
SIAM journal on applied mathematics
Mirollo and Strogatz (1990) volume: 50 issue: 6 page: 1645

When $x_i = 1$, the $i$th oscillator "fires" and $x_i$ jumps back to zero. The oscillators are assumed to interact by a simple form of pulse coupling: when a given oscillator fires, it pulls all the other oscillators up by an amount $\varepsilon$, or pulls them up to firing, whichever is less. That is,

(1.2) $$x_i(t) = 1 \Rightarrow x_j(t^+) = \min(1, x_j(t) + \varepsilon) \quad \forall j \neq i.$$

*In English this says: when the i'th oscilator reaches its peak at x(t)=1, it sends a pulse to all the other oscillators, which increase their phase by a small amount. If any of those oscillators exceed 1, they are now in sync and have the same value as the one that just fired.*

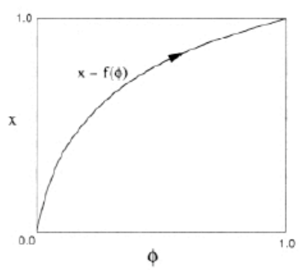



FIG. 1. *Graph of the function $f$. The timecourse of the integrate-and-fire oscillation is given by $x = f(\phi)$, where $x$ is the state and $\phi$ is a phase variable proportional to time.*

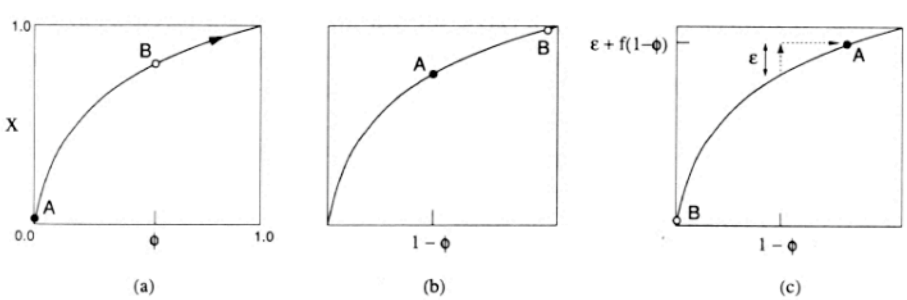



FIG. 2. *A system of two oscillators governed by $x = f(\phi)$, and interacting by the pulse-coupling rule* (1.2). (a) *The state of the system immediately after oscillator A has fired.* (b) *The state of the system just before oscillator B reaches the firing threshold. The phase difference between the oscillators is the same as in* (a). (c) *The state of the system just after B has fired. B has jumped back to zero, and the state of A is now* $\min(1, \varepsilon + f(1-\phi))$.

Some systems that can be modelled by coupled oscillators:

> Cardiovascular (heart) dynamics
> Neuronal networks (brain)
> Animal populations
> Mechanical systems like pendulums on a wall or bridges
> Social behavior (applause, attraction)

and many more. Visit the following web page for links to the articles mentioned here and lots of fun videos of fireflies, bridges, pendulums ...

http://math.ucdavis.edu/~watson/sync

you can see some of my own related research here:

http://math.ucdavis.edu/~watson

Send me email with questions, comments, ideas etc. at watson@math.ucdavis.edu