

FUNCTIONS

DEFINITION: A function is a relationship between two variables such that to each of the values of one variable (called the independent variable) there corresponds exactly one value of the other variable (called the dependent variable).

DOMAIN: the set of all values of the independent variable for which the function is defined

RANGE: the set of all values taken by the dependent variable

EXAMPLES: $c = 2\pi r$, $y = x^3 + 3$, $F = \frac{9}{5}C + 32$, $f(x) = |x+1|$, $f(x) = \sqrt{x^2 - 1}$, $f(x) = \frac{1}{x^2 + 1}$

IS y IS A FUNCTION OF x ? $\sqrt{x^2 + y = 1}$; $\text{b) } x - y^2 = 1$
 a)

THE VERTICAL LINE TEST: Graph the equation with the horizontal axis representing the independent variable. If EVERY vertical line in the plane intersects the equation at most once, then the equation defines a function.

FUNCTION NOTATION: $f(r) = 2\pi r$, $f(x) = x^3 + 3$, $f(C) = \frac{9}{5}C + 32$ $f(\cdot) = \sqrt{(\cdot)^2 - 1}$

FINDING THE DOMAIN AND RANGE OF THE FOLLOWING FUNCTIONS:

$f(x) = |x + 1|$, $f(x) = \sqrt{x^2 - 1}$, $f(x) = \frac{1}{x^2 + 1}$

DEFINITION: A function is one-to-one if to each value of dependent variable there corresponds exactly one value of the independent variable.

THE HORIZONTAL LINE TEST: Graph a function with the horizontal axis representing the independent variable. If EVERY horizontal line in the plane intersects the graph at most once, then the function is one-to-one.

COMPOSITE FUNCTION: The function given by $(f \circ g)(x) = f(g(x))$ is the composite of f with g . The domain of $(f \circ g)$ is the set of all x in the domain of g such that $g(x)$ is in the domain of f .

EXAMPLES: $f(x) = \sqrt{x - 1}$, $g(x) = x^2 + 3$, $(f \circ g)(x) = \sqrt{(x^2 + 3) - 1} = \sqrt{x^2 + 2}$,
 $(g \circ f) = (\sqrt{x - 1})^2 + 3 = x + 2$

INVERSE FUNCTION: If f and g are two functions such that $(f \circ g)(x) = x$ for each x in the domain of g , and $(g \circ f)(x) = x$ for each x in the domain of f . Then g is the inverse of f , and is denoted by $g = f^{-1}$.

EXAMPLES: $f(x) = \sqrt{x - 1}$, $f^{-1} = x^2 + 1$.

Verification: $(f \circ f^{-1}) = \sqrt{x^2 + 1 - 1} = |x| = x$ (since $x \geq 1$!!), and $(f^{-1} \circ f) = (\sqrt{x - 1})^2 + 1 = x$