Section 2.6

1. $f'(x) = -4$, $f''(x) = 0$

4. $f'(x) = 6x+4$, $f''(x) = 6$

5. $g'(t) = t^2 - 8t + 2$, $g''(t) = 2t - 8$

6. $f'(x) = 8 \cdot (x^2-1) \cdot 2x = 16x^3 - 16x$

$f''(x) = 48x^2 - 16$

9. $f'(x) = 3 \cdot 3(2-x^2)^2 \cdot (-2x) = -18x \cdot (2-x^2)^2$

$f''(x) = (-18x)' \cdot (2-x^2)^2 - 18x \cdot \left((2-x^2)^2\right)'$

$= -18 \cdot (2-x^2)^2 - 18x \cdot 2 \cdot (2-x^2) \cdot (-2x)$

$= -18 \cdot (2-x^2)^2 + 72x^2(2-x^2)$

$= (2-x^2)(-18 \times 2 + 18x^2 + 72x^2)$

$= (2-x^2)(-36 + 90x^2)$

10. $f'(x) = \left(x^{\frac{4}{3}}\right)' = \frac{4}{3} \cdot x^{\frac{1}{3}}$

$f''(x) = \frac{4}{3} \cdot \frac{1}{3} \cdot x^{-\frac{2}{3}} = \frac{4}{9} x^{-\frac{2}{3}}$

11. $f'(x) = \left(1 + 2/x-1\right)' = \left(\frac{2}{x-1}\right)' = -2 \cdot (x-1)^{-2}$

$f''(x) = -2 \cdot (-2) \cdot (x-1)^{-3} = \frac{4}{(x-1)^3}$

14. $h'(s) = 3 \cdot s^2 \cdot (s^2 - 2s+1) + s^3 \cdot (2s-2)$

$= 3s^2 (s^2 - 2s+1) + s^3 \cdot 2 \cdot (s-1)$

$= (s-1)(3s^3 - 3s^2 + 2s^3)$

$= (s-1)(5s^3 - 3s^2)$

$h''(s) = 1 \cdot (5s^3 - 3s^2) + (s-1) \cdot (15s^2 - 6s)$

$= 5s^3 - 3s^2 + 15s^3 - 6s^2 - 15s^2 + 6s$

$= 20s^3 - 24s^2 + 6s$

17. $f'_{(x)} = 5 \cdot (x+4)^3 + 5x \cdot 3(x+4)^2$

$= (5x+20)(x+4)^2 + 15x(x+4)^2$

$= (20x+20)(x+4)^2$

$f''(x) = 20 \cdot (x+4)^2 + (20x+20) \cdot 2(x+4)$

$= (x+4)(20x + 80 + 40x + 40)$

$= (x+4) \cdot (60x+120) = 60(x+4)(x+2)$

$f'''(x) = 60 \cdot (x+2) + 60 \cdot (x+4)$

$= 120x + 360$

18. $f'(x) = 2(x-1) = 2x-1$

$f''(x) = 2$

$f'''(x) = 0$

20. $f'(x) = (-1)\cdot x^{-2}$

$f''(x) = 2\cdot x^{-3}$

$f'''(x) = -6x^{-4}$

22. $f'(x) = -2x, \quad f''(x) = -2$

$f''(-\sqrt{5}) = -2$

31. $f^{(5)}(x) = 2(x+1), \quad f^{(6)}(x) = 2$

32. $f'(x) = 3x^2 - 2, \quad f''(x) = 6x$

37. $f'(x) = 1\cdot\sqrt{x^2-1} + x\cdot\frac{1}{2}(x^2-1)^{-\frac{1}{2}}\cdot 2x$

$= \sqrt{x^2-1} + \frac{x^2}{\sqrt{x^2-1}}$

$f''(x) = \frac{1}{2}(x^2-1)^{-\frac{1}{2}}\cdot 2x + \frac{2x\cdot\sqrt{x^2-1} - x^2\cdot\frac{1}{2}(x^2-1)^{-\frac{1}{2}}\cdot 2x}{x^2-1}$

$= \frac{x}{\sqrt{(x^2-1)}} + \frac{2x\cdot\sqrt{x^2-1} - x^3\cdot(x^2-1)^{-\frac{1}{2}}}{x^2-1}$

$= \frac{x\cdot(x^2-1) + 2x(x^2-1) - x^3}{(x^2-1)^{\frac{3}{2}}}$

$f''(x) = 0 \Rightarrow \quad 3x\cdot(x^2-1) - x^3 = 0$

or $\quad 3x^3 - x^3 - 3x = 0$

or $\quad 2x^3 - 3x = 0$

or $\quad x(2x^2-3) = 0 \qquad$ or $x = 0, +\sqrt{\frac{3}{2}}, -\sqrt{\frac{3}{2}}$

40. $f'(x) = \dfrac{x^2+1 - x(2x)}{(x^2+1)^2} = \dfrac{-x^2+1}{(x^2+1)^2}$

$f''(x) = \dfrac{-2x\cdot(x^2+1)^2 - (-x^2+1)\cdot 2(x^2+1)\cdot 2x}{(x^2+1)^4}$

$= \dfrac{(x^2+1)(-2x^3-2x+4x^3-4x)}{(x^2+1)^4}$

$= \dfrac{\cancel{(x^2+1)}(2x^3-6x)}{(x^2+1)^3} = \dfrac{2x(x^2-3)}{(x^2+1)^3}$

$f''(x)=0 \Rightarrow 2x\cdot(x^2-3)=0$

$\Rightarrow x=0$ or $x=\sqrt{3}$ or $x=-\sqrt{3}$.

44. $s'(t) = -16.5t + 66$

$s''(t) = -16.5$

$s'(t)=0 \Rightarrow t = \dfrac{66}{16.5} = 4$ seconds

At which time, the car stops travelling.

so $s\left(\dfrac{66}{17}\right) = -8.25 \times 4^2 + 66\times 4$

$= 132$ feet

51. False. Product rule is the correct way to differentiate a product

52. True. The polynomial is 4-th order, whose fifth order derivative will be zero.

55. True.