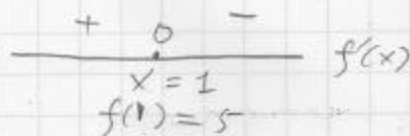


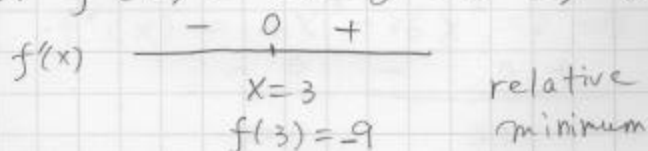
Section 3.2

1. $f'(x) = -4x + 4 = 0 \Rightarrow x = 1$

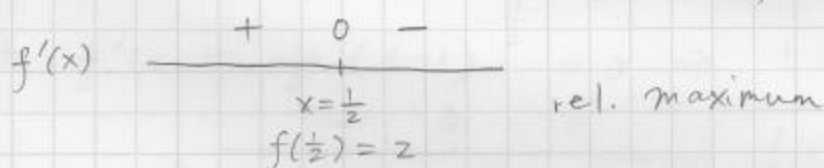
$(1, 5)$ is a relative maximum



3. $f'(x) = 2x - 6 = 0 \Rightarrow x = 3, f(3) = -9$

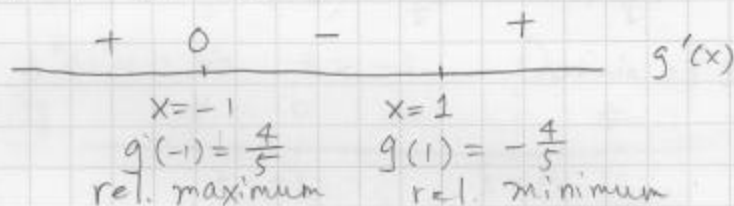


4. $f'(x) = -8x + 4 = 0 \Rightarrow x = \frac{1}{2}, f(\frac{1}{2}) = 2$

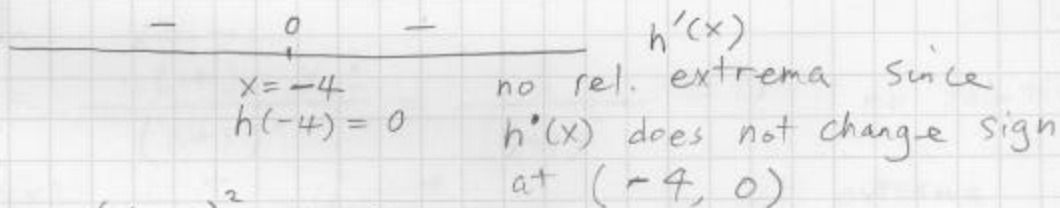


6. $g'(x) = x^4 - 1 = 0 \Rightarrow (x^2 + 1)(x + 1)(x - 1) = 0$

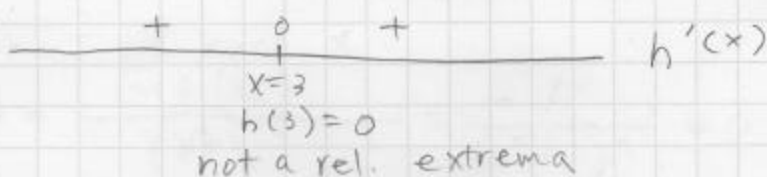
$\Rightarrow x = 1$ or $x = -1$



7. $h'(x) = -3(x+4)^2 = 0 \Rightarrow x = -4$



8. $h'(x) = 6(x-3)^2 = 0 \Rightarrow x = 3$



$$11. f'(x) = 4x^3 - 6x^2 = 0 \Rightarrow 2x^2(2x-3) = 0$$

$$\Rightarrow x=0 \text{ or } x = \frac{3}{2}$$

$$\begin{array}{c} - & 0 & - & 0 & + \\ \hline & x=0 & & x=\frac{3}{2} & \\ & f(0)=0 & & f(\frac{3}{2}) = -\frac{27}{16} & \\ \text{not rel. extrema} & & & \text{rel. minimum} & \end{array} \quad f'(x)$$

$$12. f'(x) = 4x^3 - 36x^2 = 0 \Rightarrow 4x^2(x-9) = 0 \Rightarrow x=0 \text{ or } x=9$$

$$\begin{array}{c} - & 0 & - & 0 & + \\ \hline & x=0 & & x=9 & \\ & f(0)=0 & & f(9) = -2187 & \\ \text{not rel. extrema} & & & \text{rel. minimum} & \end{array} \quad f'(x)$$

$$14. f'(t) = \frac{1}{3}(t-1)^{-\frac{2}{3}} = 0 \Rightarrow t=1$$

$$\begin{array}{c} - & 0 & + \\ \hline & t=1 & \\ & f(1) = 0 & \\ & \text{rel. minimum} & \end{array} \quad f'(t)$$

$$15. g'(t) = 1 - \frac{1}{2} \cdot (-2) \cdot \frac{1}{t^3} = 1 + \frac{1}{t^3} = \frac{t^3+1}{t^3} = 0$$

$$\Rightarrow t^3+1=0 \Rightarrow t=-1 \quad \text{Domain of } g(t): t \neq 0$$

$$\begin{array}{c} + & 0 & - & & + \\ \hline & t=-1 & & t=0 & \\ & g(-1) = -\frac{3}{2} & & & \\ & \text{rel. maximum} & & & \end{array} \quad g'(t)$$

$$17. \text{Domain: } x \neq -1$$

$$f'(x) = \frac{1 \cdot (x+1) - x \cdot 1}{(x+1)^2} = \frac{1}{(x+1)^2} = 0 \quad \text{no solution}$$

$$\begin{array}{c} f'(x) & + & & + \\ \hline & x=-1 & & \\ & \text{no rel. extrema} & & \end{array}$$

$$18. \text{Domain: all real } x$$

$$h'(x) = \frac{-4 \cdot (2x)}{(x^2+1)^2} = \frac{-8x}{(x^2+1)^2} = 0 \Rightarrow x=0$$

$$\begin{array}{c} h'(x) & + & 0 & - \\ \hline & x=0 & & \\ & h(0) = 4 & & \text{rel. maximum} \end{array}$$

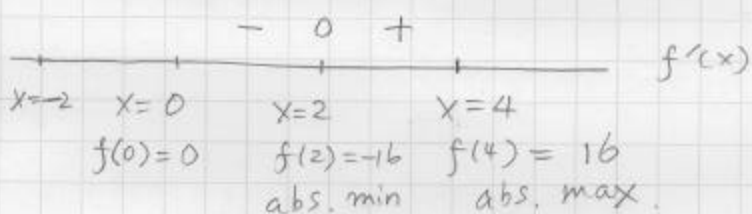
20. $f'(x) = \frac{2}{3} \neq 0$ no relative extrema on $[0, 5]$

$f(0) = \frac{5}{3}, \quad f(5) = \frac{1}{3}(10+5) = 5$

So $(0, \frac{5}{3})$ is abs minimum on $[0, 5]$

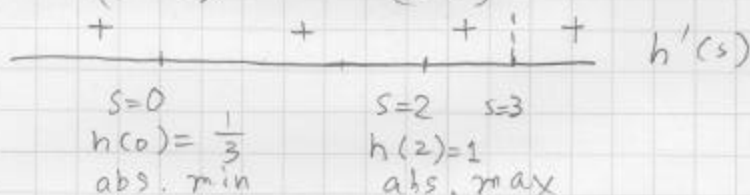
$(5, 5)$ is abs maximum on $[0, 5]$

24. $f'(x) = 3x^2 - 12 = 0 \Rightarrow 3(x+2)(x-2) = 0 \Rightarrow x = \pm 2$



25. Domain: $s \neq 3$

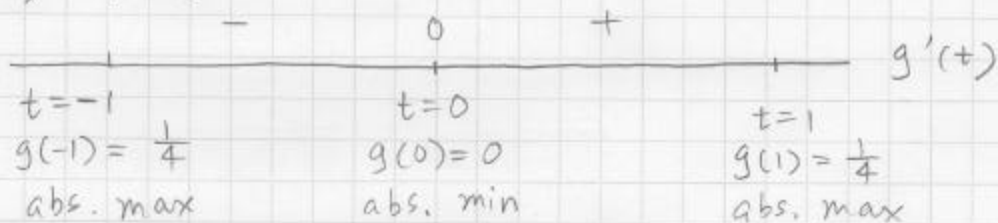
$h'(s) = \frac{+1}{(3-s)^2} = \frac{1}{(s-3)^2} = 0$ no solution



28. Domain: all real t

$g'(t) = \frac{2t \cdot (t^2+3) - t^2 \cdot (2t)}{(t^2+3)^2} = \frac{6t}{(t^2+3)^2} = 0$

$\Rightarrow t = 0$



33. Domain: $x \leq 3$

$f'(x) = \frac{4}{3} \cdot \sqrt{3-x} + \frac{4}{3} x \cdot \frac{1}{2} (3-x)^{-\frac{1}{2}} \cdot (-1)$

$= \frac{4}{3} \sqrt{3-x} - \frac{2}{3} \frac{x}{\sqrt{3-x}} = \frac{4(3-x) - 2x}{3\sqrt{3-x}} = \frac{-4x+10}{3\sqrt{3-x}} = 0$



$$x=0, f(0)=0 \rightarrow \text{abs. min}$$

$$x=\frac{5}{2}, f\left(\frac{5}{2}\right)=\frac{5}{3}\sqrt{2}, \rightarrow \text{abs. maximum}$$

$$x=3, f(3)=0 \rightarrow \text{abs. min}$$

35. Domain: all real x

$$f'(x) = \frac{4 \cdot (x^2+1) - 4x \cdot (2x)}{(x^2+1)^2} = \frac{-4x^2+4}{(x^2+1)^2} = 0$$

$$\Rightarrow -4(x^2-1)=0 \Rightarrow -4(x+1)(x-1)=0$$

$$\Rightarrow x=1 \text{ or } x=-1$$



$$\lim_{x \rightarrow \pm\infty} \frac{4x}{x^2+1} = \lim_{x \rightarrow \pm\infty} \frac{4/x}{1 + 1/x^2} = 0$$

abs maximum: $(1, 2)$

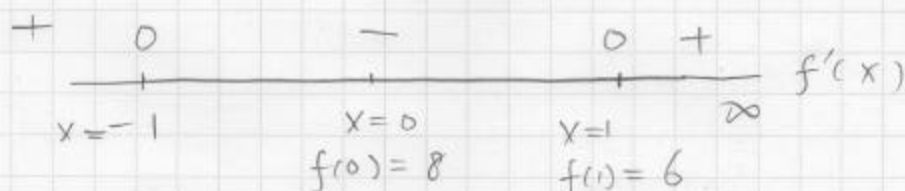
abs minimum: $(0, 0)$

horizontal asymptote: $y=0$

38. Domain: all real x

$$f'(x) = -\frac{4 \cdot (x^2+1) - 4x \cdot (2x)}{(x^2+1)^2} = -\frac{-4x^2+4}{(x^2+1)^2}$$

$$= \frac{4x^2-4}{(x^2+1)^2} = 0 \Rightarrow x=1, \text{ or } x=-1$$



$$\lim_{x \rightarrow \pm\infty} 8 \frac{4x}{x^2+1} = 8$$

abs. max: $(0, 8)$

horizontal asymptote: $y=8$

$$\begin{aligned}
 46. \quad v' &= k(-1) \cdot r^2 + k(R-r) \cdot (2r) \\
 &= -kr^2 + 2k(R-r)r \\
 &= -kr^2 + 2kRr - 2kr^2 \\
 &= -3kr^2 + 2kRr = 0
 \end{aligned}$$

$$\Rightarrow kr(-3r + 2R) = 0 \Rightarrow r=0 \text{ or } r = \frac{2R}{3}$$

(not possible)

when $r = \frac{2R}{3}$,

$$v\left(\frac{2R}{3}\right) = k \cdot \frac{R}{3} \cdot \frac{4R^2}{9} = \frac{4}{27} kR^3$$

$$\begin{array}{ccccccc}
 - & 0 & + & 0 & - & & v' \\
 \hline
 & r=0 & & r = \frac{2R}{3} & & & \\
 & & & \text{rel. max} & & &
 \end{array}$$

48. a) Year 1970 has the highest fertility rate

b) between 1985-1990, the graph has the deepest slope going upward; slowest: around 1991, 2000, and 1980

c) between 1970-1975, the decrease is the fastest
slowest: 1975-1976, 1981-1985, & around 1995.

d) demographical shifts of women's ages;
life style changes, economical reasons