Section 3.2

1. $f'(x) = -4x + 4 = 0 \Rightarrow x = 1$

$+ \quad 0 \quad -$ $f'(x)$

$x = 1$

$f(1) = 5$

$(1, 5)$ is a relative maximum

3. $f'(x) = 2x - 6 = 0 \Rightarrow x = 3, \quad f(3) = -9$

$f'(x)$ $- \quad 0 \quad +$

$x = 3$

$f(3) = -9$

relative minimum

4. $f'(x) = -8x + 4 = 0 \Rightarrow x = \frac{1}{2}, \quad f(\frac{1}{2}) = 2$

$f'(x)$ $+ \quad 0 \quad -$

$x = \frac{1}{2}$

$f(\frac{1}{2}) = 2$

rel. maximum

6. $g'(x) = x^4 - 1 = 0 \Rightarrow (x^2+1)(x+1)(x-1) = 0$

$\Rightarrow x = 1$ or $x = -1$

$+ \quad 0 \quad - \quad +$ $g'(x)$

$x = -1$ $x = 1$

$g(-1) = \frac{4}{5}$ $g(1) = -\frac{4}{5}$

rel. maximum   rel. minimum

7. $h'(x) = -3(x+4)^2 = 0 \Rightarrow x = -4$

$- \quad 0 \quad -$ $h'(x)$

$x = -4$

$h(-4) = 0$

no rel. extrema since $h'(x)$ does not change sign at $(-4, 0)$

8. $h'(x) = 6(x-3)^2 = 0 \Rightarrow x = 3$

$+ \quad 0 \quad +$ $h'(x)$

$x = 3$

$h(3) = 0$

not a rel. extrema

11. $f'(x) = 4x^3 - 6x^2 = 0 \Rightarrow 2x^2(2x-3) = 0$

$\Rightarrow x = 0$ or $x = \frac{3}{2}$

$$\begin{array}{ccccc} - & 0 & - & 0 & + \\ & x=0 & & x=\frac{3}{2} & \end{array} \quad f'(x)$$

$f(0) = 0$ $f(\frac{3}{2}) = -\frac{27}{16}$

not rel. extrema   rel. minimum

12. $f'(x) = 4x^3 - 36x^2 = 0 \Rightarrow 4x^2(x-9) = 0 \Rightarrow x=0$ or $x=9$

$$\begin{array}{ccccc} - & 0 & - & 0 & + \\ & x=0 & & x=9 & \end{array} \quad f'(x)$$

$f(0) = 0$ $f(9) = -2187$

not rel extrema   rel. minimum

14. $f'(t) = \frac{1}{3}(t-1)^{-\frac{2}{3}} = 0 \Rightarrow t = 1$

$$\begin{array}{ccc} - & 0 & + \\ & t=1 & \end{array} \quad f'(t)$$

rel. minimum

$f(1) = 0$

15. $g'(t) = 1 - \frac{1}{2}\cdot(-2)\cdot\frac{1}{t^3} = 1 + \frac{1}{t^3} = \frac{t^3+1}{t^3} = 0$

$\Rightarrow t^3+1=0 \Rightarrow t=-1$.   Domain of $g(t)$: $t \neq 0$

$$\begin{array}{ccccc} + & 0 & - & \vdots & + \\ & t=-1 & & t=0 & \end{array} \quad g'(t)$$

$g(-1) = -\frac{3}{2}$

rel. maximum

17. Domain: $x \neq -1$

$f'(x) = \frac{1\cdot(x+1) - x\cdot 1}{(x+1)^2} = \frac{1}{(x+1)^2} = 0$ no solution

$$f'(x) \quad \begin{array}{ccc} + & \vdots & + \\ & x=-1 & \end{array}$$

no rel. extrema

18. Domain: all real x

$h'(x) = \frac{-4\cdot(2x)}{(x^2+1)^2} = \frac{-8x}{(x^2+1)^2} = 0 \Rightarrow x = 0$

$$h'(x) \quad \begin{array}{ccc} + & 0 & - \\ & x=0 & \end{array}$$

$h(0) = 4$ rel. maximum

20. $f'(x) = \frac{2}{3} \neq 0$ no relative extrema on $[0, 5]$

$f(0) = \frac{5}{3}$, $f(5) = \frac{1}{3}(10+5) = 5$

So $(0, \frac{5}{3})$ is abs minimum on $[0, 5]$

$(5, 5)$ is abs maximum on $[0, 5]$

24. $f'(x) = 3x^2 - 12 = 0 \Rightarrow 3(x+2)(x-2) = 0 \Rightarrow x = \pm 2$

$-$ 0 $+$ $f'(x)$

$x=-2$ $x=0$ $x=2$ $x=4$

$f(0)=0$ $f(2)=-16$ $f(4) = 16$

abs. min abs. max

25. Domain: $s \neq 3$

$h'(s) = \frac{+1}{(3-s)^2} = \frac{1}{(s-3)^2} = 0$ no solution

$+$ $+$ $+$ $+$ $h'(s)$

$s=0$ $s=2$ $s=3$

$h(0) = \frac{1}{3}$ $h(2)=1$

abs. min abs. max

28. Domain: all real t

$g'(t) = \frac{2t \cdot (t^2+3) - t^2 \cdot (2t)}{(t^2+3)^2} = \frac{6t}{(t^2+3)^2} = 0$

$\Rightarrow t = 0$

$-$ 0 $+$ $g'(t)$

$t=-1$ $t=0$ $t=1$

$g(-1) = \frac{1}{4}$ $g(0) = 0$ $g(1) = \frac{1}{4}$

abs. max abs. min abs. max

33. Domain: $x \leq 3$

$f'(x) = \frac{4}{3} \cdot \sqrt{3-x} + \frac{4}{3}x \cdot \frac{1}{2}(3-x)^{-\frac{1}{2}} \cdot (-1)$

$= \frac{4}{3}\sqrt{3-x} - \frac{2}{3}\frac{1}{\sqrt{3-x}} = \frac{4(3-x) - 2}{3\sqrt{3-x}} = \frac{-4x+10}{3\sqrt{3-x}} = 0$

$f'(x)$ $+$ 0 $-$

0 $x = \frac{5}{2}$ 3

$x=0, \quad f(0)=0 \quad \to$ abs. min

$x=\frac{5}{2}, \quad f(\frac{5}{2}) = \frac{5}{3}\sqrt{2}, \to$ abs. maximum

$x=3, \quad f(3)=0 \quad \to$ abs. min

35. Domain: all real x

$$f'(x) = \frac{4\cdot(x^2+1) - 4x\cdot(2x)}{(x^2+1)^2} = \frac{-4x^2+4}{(x^2+1)^2} = 0$$

$$\Rightarrow \quad -4(x^2-1)=0 \Rightarrow \quad -4(x+1)(x-1)=0$$

$$\Rightarrow \quad x=1 \quad \text{or} \quad x=-1$$

$-$ 0 $+$ 0 $-$ $f'(x)$

$x=-1$ $\quad x=0$ $\quad x=1$ $\quad \infty$

$f(0)=0$ $\quad f(1)=2$

$$\lim_{x\to+\infty} \frac{4x}{x^2+1} = \lim_{x\to+\infty} \frac{4/x}{1+\frac{1}{x^2}} = 0$$

abs maximum: $(1, 2)$

abs minimum: $(0, 0)$

horizontal asymptote: $y=0$

38. Domain: all real x

$$f'(x) = -\frac{4\cdot(x^2+1) - 4x\cdot(2x)}{(x^2+1)^2} = -\frac{-4x^2+4}{(x^2+1)^2}$$

$$= \frac{4x^2-4}{(x^2+1)^2} = 0 \Rightarrow \quad x=1, \text{ or } x=-1$$

$+$ 0 $-$ 0 $+$ $f'(x)$

$x=-1$ $\quad x=0$ $\quad x=1$ $\quad \infty$

$f(0)=8$ $\quad f(1)=6$

abs. min

$$\lim_{x\to+\infty} 8 - \frac{4x}{x^2+1} = 8$$

abs. max: $(0, 8)$ $\quad$ horizontal asymptote: $y=8$

46. $v' = k(-1)\cdot r^2 + k(R-r)\cdot(2r)$

$= -kr^2 + 2k(R-r)r$

$= -kr^2 + 2kRr - 2kr^2$

$= -3kr^2 + 2kRr = 0$

$\Rightarrow \quad kr(-3r + 2R) = 0 \Rightarrow r = 0$ (not possible) or $r = \frac{2R}{3}$

when $r = \frac{2R}{3}$,

$$v\left(\frac{2R}{3}\right) = k \cdot \frac{R}{3} \cdot \frac{4R^2}{9} = \frac{4}{27} k R^3$$

$$\begin{array}{ccccccc} - & 0 & + & 0 & - & & v' \\ \hline & r=0 & & r=\frac{2R}{3} & & & \\ & & & \text{rel. max} & & & \end{array}$$

48. a) Year 1970 has the highest fertility rate

b) between 1985 — 1990, the graph has the deepest slope going upward; slowest: around 1991, 2000, and 1980

c) between 1970 — 1975, the decrease is the fastest

slowest: 1975-1976, 1981-1985, & around 1995.

d) demographical shifts of women's ages;

life style changes, economical reasons