Section 2.3

2. Imports Exports

	Imports	Exports
1980:	250	240
1990:	500	400
2000:	1220	790
2002:	1160	700

a) $\dfrac{500-250}{1990-1980} = 25$ billion

b) $\dfrac{400-240}{1990-1980} = 16$ billion

c) $\dfrac{1220-500}{2000-1990} = 72$ billion

d) $\dfrac{790-400}{2000-1990} = 39$ billion

e) $\dfrac{1160-250}{2002-1980} = \dfrac{910}{22} \approx$

f) $\dfrac{700-240}{2002-1980} = \dfrac{460}{22} \approx$

12. The medication in the bloodstream at hours

0 : ~0 1 : ~250 2 : ~550

3 : ~800 4 : ~850 5 : ~600

6 : ~0

a) So the average rates of changes are

$[0, 1]$: $\dfrac{250-0}{1} = 250$

$[1, 2]$: $\dfrac{550-250}{1} = 300$

$[2, 3]$: $\dfrac{800-550}{1} = 250$

$[3, 4]$: $\dfrac{850-800}{1} = 50$

$[4,5]$: $\dfrac{600-850}{1} = -250$

$[5,6]$: $\dfrac{0-600}{1} = -600$ milligrams/hr

The hour that the change is the greatest is between hour 5 and hour 6, and is about -600 milligrams/hour

b) The instantaneous rate change at $t=4$ is closest estimated by the ARC in $[t=3.5, t=4.5]$, for example since this interval has $t=4$ in the middle

13. $E' = \dfrac{9}{27} + \dfrac{1}{9} \times 2t - \dfrac{1}{27} \times 3t^2 = -\dfrac{t^2}{9} + \dfrac{2t}{9} + \dfrac{1}{3}$

a) $[0,1]$:

$$ARC = \frac{\frac{1}{27}(9\cdot 1 + 3\cdot 1^2 - 1^3) - \frac{1}{27}(9\cdot 0 + 3\cdot 0^2 - 0)}{1}$$

$$= \frac{11}{27}; \quad E'(0) = \frac{1}{3}, \quad E'(1) = \frac{4}{9}$$

b) $[1,2]$:

$$ARC = \frac{\frac{1}{27}(9\cdot 2 + 3\cdot 2^2 - 2^3) - \frac{1}{27}(9\cdot 1 + 3\cdot 1^2 - 1^3)}{1}$$

$$= \frac{1}{27} \times 22 - \frac{1}{27} \times 11 = \frac{11}{27}; \quad E'(2) = \frac{1}{3}$$

c) $[2,3]$:

$$ARC = \frac{\frac{1}{27}(9\cdot 3 + 3\cdot 3^2 - 3^3) - \frac{1}{27}(9\cdot 2 + 3\cdot 2^2 - 2^3)}{1}$$

$$= \frac{+27}{27} - \frac{22}{27} = \frac{5}{27}; \quad E'(3) = 0$$

d) $ARC = \dfrac{\frac{1}{27}(9\cdot 4 + 3\cdot 4^2 - 4^3) - \frac{1}{27}(9\cdot 3 + 3\cdot 3^2 - 3^3)}{1}$

$= \frac{1}{27} \times (36 + 48 - 64) - \frac{27}{27} = -\frac{7}{27}$

$E'(4) = -\frac{4^2}{9} + \frac{2\times 4}{9} + \frac{1}{3} = -\frac{5}{9}$

14. $H = 330 \cdot v^{\frac{1}{2}} - 33v + 31.35$

a) $H'(v) = \frac{d}{dv}[H] = \frac{dH}{dv} = 330 \times \frac{1}{2} \cdot v^{-\frac{1}{2}} - 33$

$= 155 v^{-\frac{1}{2}} - 33$

The rate of heat loss of the body at wind speed v is $155/\sqrt{v} - 33$ meters per second

b) $H'(2) = 155 \times 2^{-\frac{1}{2}} - 33 = \frac{155}{\sqrt{2}} - 33$

≈ 76.60

$H'(5) = 155 \times 5^{-\frac{1}{2}} - 33 \approx 36.32$

19. marginal cost is the derivative of cost

$\frac{d}{dx}[c] = 470 - 0.25 \times 2X = 470 - 0.5X$ dollars

$0 \leq X \leq 940$

20. $\frac{d}{dx}[c] = \frac{d}{dx}[900 + 300 \cdot X^{\frac{1}{2}}]$

$= 300 \cdot \frac{1}{2} \cdot X^{-\frac{1}{2}} = \frac{150}{\sqrt{X}}$ dollars

24. $\frac{d}{dx}[R] = \frac{d}{dx}[1000X - 50 \cdot X^{3/2}]$

$= 1000 - 75 \cdot X^{\frac{1}{2}}$ dollars

27. $\frac{d}{dx}[P] = \frac{d}{dx}[-0.00025x^2 + 12.2X - 25000]$

$= -0.00025 \times 2X + 12.2$

$= -0.0005X + 12.2$ dollars

32. a) $P(0) = 22 \cdot 0^2 + 52 \cdot 0 + 10000 = 10000$

$P(10) = 22 \cdot 10^2 + 52 \times 10 + 10000 = 12720$

$P(15) = 22 \times 15^2 + 52 \times 15 + 10000 = 15730$

$P(20) = 22 \times 20^2 + 52 \times 20 + 10000 = 19840$

$P(25) = 22 \times 25^2 + 52 \times 25 + 10000 = 25050$

The population of the area at 1990, 2000, 2005, 2010 are 10000, 12720, 15730, 19840 and 25050 as modeled.

34. $p'(x) = 2048 \times \frac{1}{2} \times X^{-\frac{1}{2}} - \frac{1}{8} \cdot (-2) \cdot \frac{1}{X^3}$

$= 1024/\sqrt{X} + \frac{1}{4} \cdot \frac{1}{X^3}$

a) $p'(150) = \frac{1024}{\sqrt{150}} + \frac{1}{4} \cdot \frac{1}{150^3} \approx 83.61$

b) $p'(175) = \frac{1024}{\sqrt{175}} + \frac{1}{4} \cdot \frac{1}{175^3} \approx 77.41$

c) $p'(200) = \frac{1024}{\sqrt{200}} + \frac{1}{4} \cdot \frac{1}{200^3} \approx 72.41$

d) $p'(225) = \frac{1024}{\sqrt{225}} + \frac{1}{4} \cdot \frac{1}{225^3} \approx 68.27$

e) $p'(250) = 1024/\sqrt{250} + \frac{1}{4} \cdot \frac{1}{250^3} \approx 64.76$

f) $p'(275) = \frac{1024}{\sqrt{275}} + \frac{1}{4} \cdot \frac{1}{275^3} \approx 61.75$

37. a) Let q be the price each lemonade is sold for, then the linear relation between the # of units sold (x) to q is

$$x - 400 = m(q - 0.75)$$

with $m = \frac{500 - 400}{0.50 - 0.75} = -400$

so the equation for demand (x) and price is:

$$x = 400 - 400(q - 0.75) = -400q + 700$$

or

$$q = \frac{1}{400}(700 - x)$$

and revenue is

$$R = q \cdot x = \frac{1}{400}(700 - x) \cdot x$$
$$= \frac{-x^2}{400} + \frac{7x}{4}$$

and cost for x units is

$$C = 0.05x + 20$$

Therefore, profit is

$$P = R - C = -\frac{x^2}{400} + \frac{7}{4}x - 0.05x - 20$$
$$= -\frac{x^2}{400} + 1.70x - 20$$

b) $p'(x) = -\frac{2x}{400} + 1.70 = -\frac{1}{200}x + 1.70$

$p'(200) = -1 + 1.7 = +0.7 > 0$

$p'(400) = -\frac{400}{200} + 1.7 = -0.3 < 0$

c) $p'(200) = 0.7$ dollars (profit)

$p'(400) = -0.3$ dollar (loss)

49. rate of growth increases in lag phase, accelerates in acceleration phase, decelerates in deceleration phase and reduces to zero in equilibrium phase.