16A Section 3 Exam 1
Oct 15, 2010

Your Name:

Your Student ID:

1.) It is a violation of the university honor code to, in any way, assist another person in the completion of this exam, copy answers from another student's exam, have another student take your exam for you. Please keep your own work covered up as much as possible during the exam so that others will not be tempted or distracted. Thank you for your cooperation.
2.) No notes, books, or classmates may be used as resources for this exam. YOU MAY NOT USE A CALCULATOR ON THIS EXAM.
3.) Using only a calculator to determine the value of limits will receive little credit.
4.) You will be graded on proper use of limit notation.
5.) Read directions to each problem carefully. Show all work for full credit. In most cases, a correct answer with no supporting work will NOT receive full credit. What you write down and how you write it are the most important means of your getting a good score on this exam. Neatness and organization are also important.

Problem	1	2	3	4	5	Bonus	Total
Points	20	20	18	12	30	10	100 + 10
Score							

Problem 1 [20 points]

(a) [5 points] Find the distance between the points $(\frac{1}{2}, 1)$ and $(\frac{11}{2}, -11)$

$$d = \sqrt{\left(\frac{1}{2} - \frac{11}{2}\right)^2 + (1 - (-11))^2} = \sqrt{(-5)^2 + 12^2} = 13$$

(b) [5 points] Find the coordinates of the mid-point of the points $(\frac{1}{2}, 1)$ and $(\frac{11}{2}, -11)$

$$(x, y) = \left(\frac{\frac{1}{2} + \frac{11}{2}}{2}, \frac{1-11}{2}\right) = (3, -5)$$

(c) [5 points] Suppose that $(\frac{1}{2}, 1)$ and $(\frac{11}{2}, -11)$ are two ends of a diameter of a circle. Find the equation for this circle. (Hint: find the center and radius of the circle first)

circle is centered at $(3, -5)$, with radius 13, so the equation is

$$(x-3)^2 + (y+5)^2 = \left(\frac{13}{2}\right)^2$$

(d) [5 points] Write the equation of the line that passes through (1,2) and has a slope of -4. (You don't need to simply the equation)

slope: $m = -4$

equation: $\dfrac{y-2}{x-1} = -4$ or $y - 2 = -4(x-1)$

or $y = -4x + 6$

Problem 2 [20 points]

(a) [5 points] Find the domain of the function $f(x) = \sqrt{2x-5}$

domain: $2x - 5 \geq 0$

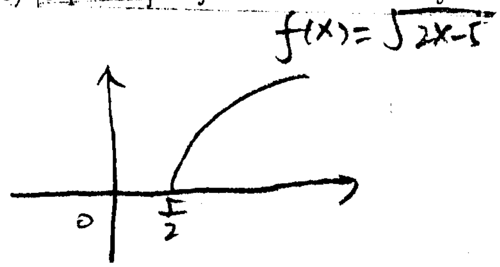
$\Leftrightarrow x - \frac{5}{2} \geq 0$

$\Leftrightarrow x \geq \frac{5}{2}$

(b) [5 points] Find the range of the function $f(x) = \sqrt{2x-5}$

range: $y \geq 0$

(c) [5 points] Is f one-to-one? Why or why not?



f is one-to-one, since it passes the horizontal test,

(d) [5 points] Let f be defined as in part (a). Find a function g such that $f(g(x)) = x$ (Hint: inverse function)

$y = \sqrt{2x-5}$

change variables: $x = \sqrt{2y-5}$

$\Leftrightarrow x^2 = 2y - 5$

solve for y:

$2y = x^2 + 5$

$\Leftrightarrow y = \frac{1}{2}(x^2+5)$

Problem 3 [18 points]

Find the limits (if possible) (a) [6 points]

$$\lim_{x\to 1} \frac{\frac{1}{x+4} - \frac{1}{4}}{x}$$

(Hint: adding the fractions up in the numerator)

$$= \lim_{x\to 1} \frac{\frac{4-(x+4)}{4(x+4)}}{x} = \lim_{x\to 1} \frac{\frac{-x}{4(x+4)}}{x} = \lim_{x\to 1} \frac{-1}{4(x+4)}$$

$$= \frac{-1}{4\cdot 5} = -\frac{1}{20}$$

(b) [6 points]

$$\lim_{x\to 0^+} 1 + \frac{1}{x} = +\infty$$

(c) [6 points]

$$\lim_{x\to 3} \frac{\sqrt{x+1}+1}{x}$$

$$= \frac{\sqrt{3+1}+1}{3} = \frac{2+1}{3} = \frac{3}{3} = 1$$

Problem 4 [12 points]

(a) [6 points] Use the limit definition of derivative to find the derivative of $f(x) = \frac{1}{2x+3}$ at $(1, \frac{1}{5})$.

$$f'(1) = \lim_{\Delta x \to 0} \frac{f(1+\Delta x) - f(1)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{1}{2(1+\Delta x)+3} - \frac{1}{2+3}}{\Delta x} = \lim_{\Delta x \to 0} \frac{\frac{1}{5+2\Delta x} - \frac{1}{5}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{5-5-2\Delta x}{5(5+2\Delta x)}}{\Delta x} = \lim_{\Delta x \to 0} \frac{\frac{-2\Delta x}{5(5+2\Delta x)}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-2}{5(5+2\Delta x)} = \frac{-2}{25}$$

(b) [6 points] Use the limit definition of derivative to find the derivative of $f(t) = t^3 + 12t$.

$$f'(t) = \lim_{\Delta t \to 0} \frac{[(t+\Delta t)^3 + 12(t+\Delta t)] - (t^3 + 12t^3)}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \left[\frac{[t^3 + 3t^2h + 3th^2 + h^3 + 12t - 12\Delta t]}{\Delta t} - \frac{t^3 + 12t}{\Delta t} \right]$$

$$= \lim_{\Delta t \to 0} \frac{3t^2\Delta t + 3t(\Delta t)^2 + (\Delta t)^3 + 12\Delta t}{\Delta t}$$

$$= \lim_{\Delta t \to 0} 3t^2 + 3t(\Delta t) + (\Delta t)^2 + 12 = 3t^2 + 12$$

Problem 5 [30 points]

Find the derivatives of the following functions.

(a) [4 points] $f(x) = 2$

$$f'(x) = 0$$

(b) [4 points] $f(x) = (x+1)(3x-1)$

$$f(x) = 3x^2 + 3x - x - 1 = 3x^2 + 2x - 1$$

$$f'(x) = 6x + 2$$

(c) [4 points] $f(x) = \frac{2}{x+1}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{\frac{2}{x+\Delta x+1} - \frac{2}{x+1}}{\Delta x \to 0} = \lim_{\Delta x \to 0} \frac{\frac{2x+2-2x-2\Delta x-2}{(x+\Delta x+1)(x+1)}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{-2\Delta x}{(x+\Delta x+1)(x+1)}}{\Delta x} = \lim_{\Delta x \to 0} \frac{-2}{(x+\Delta x+1)(x+1)} = \frac{-2}{(x+1)^2}$$

(d) [4 points] $f(x) = 3x^3 + 2x - 4$

$$f'(x) = 9x^2 + 2$$

(e) [4 points] $f(x) = 5x^{-3}$

$$f'(x) = 5 \cdot (-3) \cdot x^{-4} = -15x^{-4}$$

Bonus Problem [OPTIONAL]

The problems below worths 10 points. This problem is optional.

Find the derivative of $f(x) = (3x^2 - 2)(x^3 + 5)$

$$f(x) = 3x^5 + 15x^2 - 2x^3 - 10$$

$$= 3x^5 - 2x^3 + 15x^2 - 10$$

$$f'(x) = 15x^4 - 6x^2 + 30x$$