



**Problem 1**

[10 points]

Find the limits below.

(a) [5 points]

$$\lim_{x \rightarrow 0} \frac{\sqrt{x+1} - 1}{x} =$$

(b) [5 points]

$$\lim_{x \rightarrow 0} \frac{x^2}{x} =$$

**Problem 2**

[10 points]

(a) [5 points] Solve the following trigonometry equation for  $0 \leq \theta \leq 2\pi$ :

$$2 \cos^2 \theta - \cos \theta = 1$$

(b) [5 points] Use the limit definition of derivative to compute the derivative of  $f(x) = x^2$ .

**Problem 3**

[20 points]

Find the derivative  $\frac{dy}{dx}$  for each of the following equation.

(a) [5 points]  $y = \frac{1}{x^2+1}$

(b) [5 points]  $y = \sin(2x^2)$

(c) [5 points]  $x^2 + y^2 = 4$

(d) [5 points]

$$\frac{xy - y^2}{y - x} = 4$$

**Problem 4**

[18 points]

(a) [6 points] Find the differential  $dy$  for the following function.

$$y = -(x - 1)^3(x - 5)$$

(b) [6 points] Find the vertical and horizontal asymptotes of

$$f = \frac{x^2 - 2x - 24}{x^2 + x - 12}$$

(c) [6 points] Find the second derivative of  $y = x^2 \cos(3x)$

**Problem 5**

[12 points]

(a) [6 points] The surface area of a cube is increasing at a rate of  $4 \text{ in}^2/\text{sec}$ . How fast is the volume changing when the length of each edge is 2 inches long?

(b) [6 points] Find the equation for the line that pass through the point  $(0, 0)$  and is parallel to the tangent of  $f(x) = x^2$  at  $x = 1/2$ .

**Problem 6**

[10 points]

The profit derived from selling  $x$  units of a particular product is modeled by the formula

$$P = 16x^2 + 30x - 170.$$

(a) [4 points] Find the formula for the differential  $dP$ .

(b) [3 points] What is the actual gain in profit obtained by increasing the sales from 20 to 21 units?

(c) [3 points] Use the differential  $dP$  to approximate the change in profit when the sales increase from 20 to 21 units.

**Problem 7 (Multiple-Choices/True-and-False Questions)**

[20 points]

Please circle one and only one answer for problems 1-5.

(1)

$$\lim_{x \rightarrow \pi^-} \frac{\cos x}{x} =$$

- (a)
- $-\pi$
- (b)
- $1/\pi$
- (c)
- $\infty$
- (d) Do not exist    (e) None of the above

(2) The domain of  $f(x) = 1/\sqrt{1-x^2}$  is

- (a) all real numbers    (b) all real numbers except for
- $x = 1$
- or
- $x = -1$
- 
- (c)
- $-1 \leq x \leq 1$
- (d)
- $-1 < x < 1$

(3) If  $y = \sin^2(4x)$  then  $y'(\frac{\pi}{12}) =$ 

- (a) 8    (b) 0    (c)
- $4\sqrt{3}$
- (d)
- $2\sqrt{2}$
- (e) None of the above

(4)

$$\lim_{x \rightarrow -3} \frac{x^2 - 9}{x + 3} =$$

- (a)
- $\infty$
- (b) 2    (c)
- $-\infty$
- (d)
- $-2$
- (e) None of the above

(5) The curve  $y = x^2 + 10x$  has a horizontal tangent line when  $x$  is

- (a) 5    (b)
- $-5$
- (c) 10    (d)
- $-10$
- (e) None of the above

**Please read carefully each statement in Problems 6-10, and determine whether each is True or False.**(6) The product rule is  $(f \cdot g)' = f' \cdot g + f \cdot g'$ .

- 
- True
- 
- False

(7) If  $f(x) = x^2 + 1$  and  $g(x) = \sqrt{x}$ . Then  $f(g(x)) = x + 1$ .

- 
- True
- 
- False

(8) If  $(c, f(c))$  is a point of inflection of the graph of  $f$ , then the derivative of  $f$  must be zero.

- 
- True
- 
- False

(9)  $y = x^3$  has an inflection point at  $(0, 0)$ .

- 
- True
- 
- False

(10) The second derivative of  $f(x) = \cos(x)$  is  $f''(x) = \tan(x)$ .

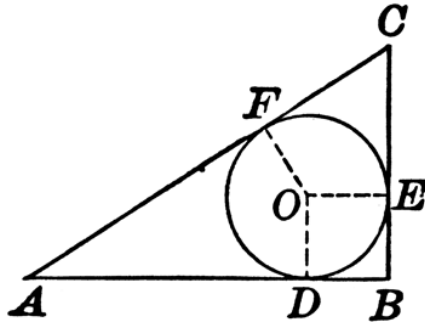
- 
- True
- 
- False

**Bonus Problems**

[20 points]

The following problems are optional and are worth 10 points each.

1.) A circle is inscribed in the right triangle shown below. We know that the length of the hypotenuse is  $\overline{AC} = 1$ , and the length of the shortest edge is  $\overline{BC} = \frac{1}{2}$ . What is the radius ( $\overline{OE}$ ,  $\overline{OF}$ , or  $\overline{OD}$ ) of the circle?



2.) A square is inscribed in the given right triangle. Find the area of the square.

