

Problem 1

[10 points]

Find the limits below.

(a) [5 points]

$$\lim_{x \rightarrow 0} \frac{\sqrt{x+1} - 1}{x} =$$

(b) [5 points]

$$\lim_{x \rightarrow 0} \frac{x^2}{x} =$$

Problem 2

[10 points]

(a) [5 points] Solve the following trigonometry equation for $0 \leq \theta \leq 2\pi$:

$$2 \cos^2 \theta - \cos \theta = 1$$

(b) [5 points] Use the limit definition of derivative to compute the derivative of $f(x) = x^2$.

Problem 3

[20 points]

Find the derivative $\frac{dy}{dx}$ for each of the following equation.

(a) [5 points] $y = \frac{1}{x^2+1}$

(b) [5 points] $y = \sin(2x^2)$

(c) [5 points] $x^2 + y^2 = 4$

(d) [5 points]

$$\frac{xy - y^2}{y - x} = 4$$

Problem 4

[18 points]

(a) [6 points] Find the differential dy for the following function.

$$y = -(x - 1)^3(x - 5)$$

(b) [6 points] Find the vertical and horizontal asymptotes of

$$f = \frac{x^2 - 2x - 24}{x^2 + x - 12}$$

(c) [6 points] Find the second derivative of $y = x^2 \cos(3x)$

Problem 5

[12 points]

(a) [6 points] The surface area of a cube is increasing at a rate of $4 \text{ in}^2/\text{sec}$. How fast is the volume changing when the length of each edge is 2 inches long?

(b) [6 points] Find the equation for the line that pass through the point $(0, 0)$ and is parallel to the tangent of $f(x) = x^2$ at $x = 1/2$.

Problem 6

[10 points]

The profit derived from selling x units of a particular product is modeled by the formula

$$P = 16x^2 + 30x - 170.$$

(a) [4 points] Find the formula for the differential dP .

(b) [3 points] What is the actual gain in profit obtained by increasing the sales from 20 to 21 units?

(c) [3 points] Use the differential dP to approximate the change in profit when the sales increase from 20 to 21 units.

Problem 7 (Multiple-Choices/True-and-False Questions)

[20 points]

Please circle one and only one answer for problems 1-5.

(1)

$$\lim_{x \rightarrow \pi^-} \frac{\cos x}{x} =$$

- (a)
- $-\pi$
- (b)
- $1/\pi$
- (c)
- ∞
- (d) Do not exist (e) None of the above

(2) The domain of $f(x) = 1/\sqrt{1-x^2}$ is

- (a) all real numbers (b) all real numbers except for
- $x = 1$
- or
- $x = -1$
-
- (c)
- $-1 \leq x \leq 1$
- (d)
- $-1 < x < 1$

(3) If $y = \sin^2(4x)$ then $y'(\frac{\pi}{12}) =$

- (a) 8 (b) 0 (c)
- $4\sqrt{3}$
- (d)
- $2\sqrt{2}$
- (e) None of the above

(4)

$$\lim_{x \rightarrow -3} \frac{x^2 - 9}{x + 3} =$$

- (a)
- ∞
- (b) 2 (c)
- $-\infty$
- (d)
- -2
- (e) None of the above

(5) The curve $y = x^2 + 10x$ has a horizontal tangent line when x is

- (a) 5 (b)
- -5
- (c) 10 (d)
- -10
- (e) None of the above

Please read carefully each statement in Problems 6-10, and determine whether each is True or False.(6) The product rule is $(f \cdot g)' = f' \cdot g + f \cdot g'$.

-
- True
-
- False

(7) If $f(x) = x^2 + 1$ and $g(x) = \sqrt{x}$. Then $f(g(x)) = x + 1$.

-
- True
-
- False

(8) If $(c, f(c))$ is a point of inflection of the graph of f , then the derivative of f must be zero.

-
- True
-
- False

(9) $y = x^3$ has an inflection point at $(0, 0)$.

-
- True
-
- False

(10) The second derivative of $f(x) = \cos(x)$ is $f''(x) = \tan(x)$.

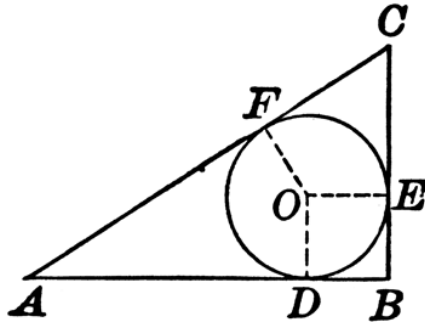
-
- True
-
- False

Bonus Problems

[20 points]

The following problems are optional and are worth 10 points each.

- 1.) A circle is inscribed in the right triangle shown below. We know that the length of the hypotenuse is $\overline{AC} = 1$, and the length of the shortest edge is $\overline{BC} = \frac{1}{2}$. What is the radius (\overline{OE} , \overline{OF} , or \overline{OD}) of the circle?



- 2.) A square is inscribed in the given right triangle. Find the area of the square.

