

Math 21A
 Vogler
 Discussion Sheet 3

1.) Evaluate the following limits.

a.) $\lim_{x \rightarrow 4} \frac{\sqrt{5+x} - 3}{\sqrt{x} - 2}$

b.) $\lim_{x \rightarrow \infty} (x - x \cos(4/x))$

c.) $\lim_{x \rightarrow 0} \frac{2x}{\sin x - x}$ (HINT : $\frac{\sin x}{x} \leq 1$.)

2.) Use one-sided limits and limits to infinity to find all vertical and horizontal asymptotes for the following functions. Use the asymptotes together with x- and y-intercepts to sketch graphs of each function.

a.) $y = \frac{x-2}{x^2-9}$

b.) $y = \frac{x+2}{x^2(x+1)}$

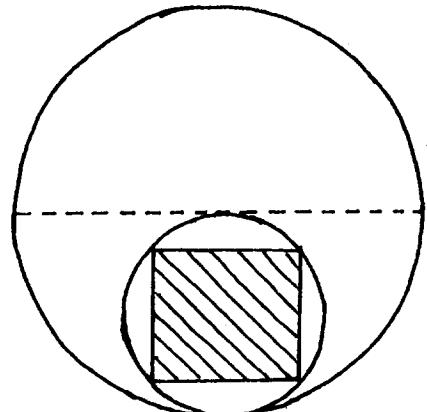
3.) Give an ϵ, δ -proof for $\lim_{x \rightarrow 1} \frac{x+3}{1+\sqrt{x}} = 2$.

4.) Use the IMVT to determine if the following equation is solvable. This is a writing exercise : $x^3 + x - \sqrt{x+4} = 0$.

5.) In the given diagram the smaller circle is the largest one that can be inscribed in the given semi-circle. If the larger circle has circumference 4π in., what is the area of the inscribed shaded square ?

6.) Determine the x-values for which the following function is continuous. It is not necessary to graph the function :

$$f(x) = \begin{cases} \frac{\sin 3x}{x}, & \text{if } x < 0 \\ 3.01, & \text{if } x = 0 \\ \frac{x-1}{\sqrt{x-1}}, & \text{if } 0 < x < 1 \\ 2, & \text{if } x \geq 1. \end{cases}$$



7.) Use the limit definition of derivative,

$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$, to differentiate each of the following functions.

a.) $f(x) = \cos x$

b.) $f(x) = \frac{7+x}{3x-5}$

c.) $f(x) = \sqrt{x^2 + x}$

8.) Let $f(x) = \begin{cases} \sin 2x, & \text{if } x \geq 0 \\ 2x, & \text{if } x < 0 \end{cases}$. Use the limit definition of derivative to determine $f'(0)$

++++++

The following problem is for recreational purposes only.

9.) Without lifting your pencil, join all sixteen dots with six straight lines.

