Math 21A
Vogler
Discussion Sheet 8

1.) Consider the function $f(x) = x^{3/2} + x^{1/2}$. Show that f satisfies the assumptions of the Mean Value Theorem (MVT) on the interval $[0, 1]$, and determine all values of c guaranteed by the MVT.

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2.) Explain why $f(x) = x^3 - x^{1/3}$ does not satisfy all of the assumptions of the MVT on the interval $[-1, 1]$.

3.) Consider the function $f(x) = \begin{cases} x^2 , & \text{if } 0 \leq x \leq 3 \\ x^3 , & \text{if } -1 \leq x < 0 \end{cases}$.

a.) Sketch the graph of f .

b.) Show that f satisfies the conditions of the Mean Value Theorem (MVT) over the interval [-1, 3], including special attention at $x = 0$, and determine all values of c guaranteed by the MVT.

4.) Determine the domain of each function and where each function is increasing, decreasing, concave up, and concave down. Identify all relative and absolute extrema, inflection points, x- and y-intercepts, and asymptotes (vertical, horizontal, or tilted). Sketch the graph.

a.) $f(x) = x(5 - x)^4$

b.) $y = \dfrac{x}{x^2 + 4}$

c.) $g(x) = \dfrac{x^2 + 4}{x}$

d.) $f(x) = x^{2/3} + x$ on the interval $[-1, 8]$

e.) $f(x) = \sqrt{3} \sin x + \cos x$ on the interval $[0, 2\pi]$

8.) Write a formula for a function f which has the following five properties.

a.) f has exactly one horizontal asymptote.
b.) f has exactly two vertical asymptotes.
c.) f has at least two x-intercepts.
d.) f has at least one relative (or absolute) extrema.
e.) f has at least one point of inflection.

9.) Consider the function $f(x) = 1 - x^{2/3}$ on the interval [-1, 1] . Show that $f(1) = f(-1) = 0$, but that $f'(x)$ is never zero on the interval [-1, 1]. Explain how this is possible, in view of Rolle's Theorem.

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The following problem is for recreational purposes only.

11.) Assume that you have three boxes labeled and filled with fruit. One box contains APPLES only. One box contains ORANGES only. One box contains APPLES and ORANGES. Unfortunately, ALL of the boxes are labeled incorrectly. Explain how to correctly relabel all of the boxes by (without peeking into any box) selecting exactly one fruit from exactly one box.