1.) Determine the following limits.

a.) $\lim\limits_{x \to 2} \dfrac{x-2}{x^2+x-6}$ $= \text{"}\frac{0}{0}\text{"} = \lim\limits_{x \to 2} \dfrac{\cancel{(x-2)}}{\cancel{(x-2)}(x+3)} = \dfrac{1}{5}$

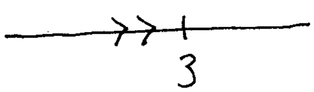
b.) $\lim\limits_{x \to 0} \dfrac{\frac{1}{x+1} - 1}{\frac{x}{1}}$ $= \text{"}\frac{0}{0}\text{"} = \lim\limits_{x \to 0} \dfrac{1-(x+1)}{x+1} \cdot \dfrac{1}{x}$

$= \lim\limits_{x \to 0} \dfrac{-\cancel{x}}{(x+1)\cancel{x}} = \dfrac{-1}{1} = -1$

c.) $\lim\limits_{h \to 0} \dfrac{\sin h^2}{h}$ $= \lim\limits_{h \to 0} \dfrac{h}{h} \cdot \dfrac{\sin h^2}{h} = \lim\limits_{h \to 0} h \cdot \dfrac{\sin h^2}{h^2}$

$= 0 \cdot 1 = 0$

d.) $\lim\limits_{x \to +3^-} \dfrac{x^2-5}{3-x}$ $= \dfrac{4}{0^+} = +\infty$



e.) $\lim\limits_{x \to 1} \dfrac{2-\sqrt{x+3}}{x-1}$ $= \text{"}\frac{0}{0}\text{"} = \lim\limits_{x \to 1} \dfrac{2-\sqrt{x+3}}{x-1} \cdot \dfrac{2+\sqrt{x+3}}{2+\sqrt{x+3}}$

$= \lim\limits_{x \to 1} \dfrac{4-(x+3)}{(x-1)(2+\sqrt{x+3})} = \lim\limits_{x \to 1} \dfrac{-\cancel{(x-1)}}{\cancel{(x-1)}(2+\sqrt{x+3})} = \dfrac{-1}{4}$

f.) $\lim\limits_{x \to \infty} \dfrac{\cos(3x+1)}{3x+1}$ (HINT : Use the Squeeze Principle.) $-1 \le \cos(3x+1) \le +1$

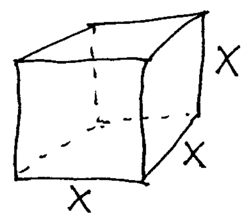
$\rightarrow \dfrac{-1}{3x+1} \le \dfrac{\cos(3x+1)}{3x+1} \le \dfrac{+1}{3x+1}$ and $\lim\limits_{x \to \infty} \dfrac{-1}{3x+1} = 0 = \lim\limits_{x \to \infty} \dfrac{1}{3x+1}$

so $\lim\limits_{x \to \infty} \dfrac{\cos(3x+1)}{3x+1} = 0$

2.) Determine the domain for $f(x) = \dfrac{3}{4 - \sqrt{x}}$.

$x \geq 6$ and $\sqrt{x} \neq 4$ so $x \neq 16$ so

Domain : all $x \geq 0$ except $x = 16$



3.) Consider a three-dimensional cube with side length x.

a.) Write the volume V of the cube as a function of x.

$V = x^3$

b.) Write the surface area S of the cube as a function of x.

$S = 6x^2$

c.) Write the surface area S of the cube as a function of the volume V.

$V = x^3 \rightarrow x = V^{1/3} \rightarrow$

$S = 6x^2 = 6\left(V^{1/3}\right)^2 = 6V^{2/3}$.

4.) Consider the following function $f(x) = \begin{cases} \dfrac{x^2-3x}{x^2-9}, & \text{if } x \neq 3, -3 \\ \frac{1}{2}, & \text{if } x = 3 \\ 0, & \text{if } x = -3 . \end{cases}$

Determine if f is continuous at $x = 3$.

i.) $f(3) = 1/2$

ii.) $\lim_{x \to 3} f(x) = \lim_{x \to 3} \dfrac{x^2-3x}{x^2-9} \overset{\frac{0}{0}}{=} \lim_{x \to 3} \dfrac{x\cancel{(x-3)}}{\cancel{(x-3)}(x+3)}$

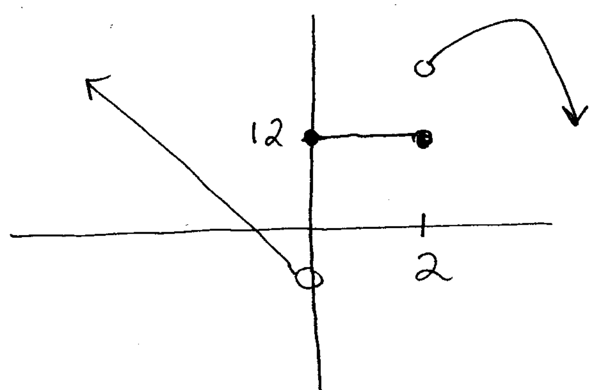
$= 3/6 = 1/2$

iii.) $\lim_{x \to 3} f(x) = f(3)$ so

f is continuous at $x = 3$.

5.) Using limits, determine the value(s) of constants A and B so that the following function is continuous for all values of x :

$$f(x) = \begin{cases} Ax + B, & \text{if } x < 0 \\ 12 , & \text{if } 0 \le x \le 2 \\ Bx^2 - A, & \text{if } x > 2 . \end{cases}$$



$\underline{\lim_{x \to 0^-} (Ax+B) = 12}$ and $\underline{\lim_{x \to 2^+} (Bx^2 - A) = 12}$

$\to$ $B = 12$ and $4B - A = 12 \to$

$48 - A = 12 \to$ $A = 36$

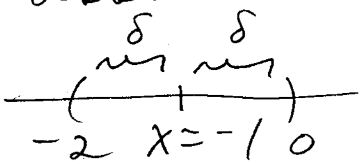
6.) Use the Intermediate Value Theorem to prove that the equation $x^3 = x^2 + 5$ is solvable. This is a writing exercise.

Since $x^3 - x^2 = 5$ let $f(x) = x^3 - x^2$ and $m = 5$, where f is continuous for all x-values since it is a polynomial. Since $f(0) = 0$ and $f(3) = 18$ and $m = 5$ is between $f(0)$ and $f(3)$, choose interval $[0, 3]$. By the IMVT there is at least one number c, $0 \le c \le 3$, so that $f(c) = m$, i.e., $c^3 - c^2 = 5$ and equation is solvable.

7.) Give an ε, δ-proof for the following limit. This is a writing exercise. :
$\lim_{x \to -1} (x^2 + 3) = 4$

Let $\varepsilon > 0$ be given. Find $\delta > 0$ so that if $0 < |x+1| < \delta$, then $|f(x) - 4| < \varepsilon$. Begin with $|f(x) - 4| < \varepsilon$ and solve for $|x+1|$.

Then $|f(x) - 4| < \varepsilon$
iff $|(x^2 + 3) - 4| < \varepsilon$
iff $|x^2 - 1| < \varepsilon$
iff $|x-1||x+1| < \varepsilon$.
Replace $|x-1|$.
Assume $\delta \le 1$



so that $-2 < x < 0$ and $1 < |x-1| < 3$. Then $|x-1||x+1| < 3|x+1| < \varepsilon$ iff $3|x+1| < \varepsilon$ iff $|x+1| < \varepsilon/3$.
Choose $\delta = \min\{\varepsilon/3, 1\}$.
Thus, if $0 < |x+1| < \delta$, it follows that $|f(x) - 4| < \varepsilon$.

2.) Use $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ to differentiate the function $f(x) = \frac{x}{x+5}$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{x+h}{x+h+5} - \frac{x}{x+5}}{\frac{h}{1}}$$

$$= \lim_{h \to 0} \frac{(x+h)(x+5) - x(x+h+5)}{(x+h+5)(x+5)} \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + \cancel{5x} + \cancel{hx} + 5h - \cancel{x^2} - \cancel{hx} - \cancel{5x}}{(x+h+5)(x+5) \cdot h}$$

$$= \lim_{h \to 0} \frac{5\cancel{h}}{(x+h+5)(x+5)\cancel{h}} = \frac{5}{(x+5)^2}$$

3.) You are standing on the top edge of a building which is 96 ft. high. You throw an apple straight UP at 80 ft./sec. and watch as it falls back to the ground.

a.) Assume that the acceleration due to gravity is $s''(t) = -32$ ft./sec.^{2}. Derive velocity, $s'(t)$, and height (above ground), $s(t)$, formulas for this apple.

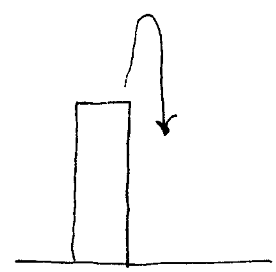
$s''(t) = -32 \rightarrow$

$s'(t) = -32t + c$ (and $s'(0) = 80$ ft./sec.)

$\rightarrow c = 80 \rightarrow \boxed{s'(t) = -32t + 80}$;

$s(t) = -16t^2 + 80t + c$ (and $s(0) = 96$ ft.)

$\rightarrow c = 96 \rightarrow \boxed{s(t) = -16t^2 + 80t + 96}$



b.) In how many seconds will the apple strike the ground ?

strike ground : $s(t) = 0 \rightarrow$

$-16t^2 + 80t + 96 = 0 \rightarrow -16(t^2 - 5t - 6) = 0 \rightarrow$

$-16(t-6)(t+1) = 0 \rightarrow$ $t = 6$ sec.

c.) How high does the apple go ?

highest point: $s'(t) = 0 \rightarrow -32t + 80 = 0 \rightarrow$

$\boxed{t = 2.5 \text{ sec.}} \rightarrow$

$s(2.5) = -16(2.5)^2 + 80(2.5) + 96 = $ 196 ft.

The following EXTRA CREDIT PROBLEM is worth   points. This problems is OPTIONAL.

1.) Determine the following limit : $\lim\limits_{x \to \infty} (x - \sqrt{x^2 + 9x}\ ) = \infty - \infty$

$$\lim_{x \to \infty} \left(x - \sqrt{x^2+9x}\right) \frac{\left(x + \sqrt{x^2+9x}\right)}{\left(x + \sqrt{x^2+9x}\right)}$$

$$= \lim_{x \to \infty} \frac{x^2 - (x^2 + 9x)}{x + \sqrt{x^2+9x}}$$

$$= \lim_{x \to \infty} \frac{-9x}{x + \sqrt{x^2+9x}} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}$$

$$= \lim_{x \to \infty} \frac{-9}{1 + \sqrt{x^2+9x} \cdot \sqrt{\frac{1}{x^2}}}$$

$$= \lim_{x \to \infty} \frac{-9}{1 + \sqrt{1 + 9/x}} =$$

$$= \frac{-9}{1 + \sqrt{1+0}}$$

$$= -\frac{9}{2}.$$