1.) Differentiate each of the following functions. DO NOT SIMPLIFY ANSWERS.

a.) $y = \pi + (5x+1)^{-4}$

$$Y' = -4(5X+1)^{-5} \cdot 5$$

b.) $f(x) = \sec x \cdot \tan 3x$

$$f'(x) = \sec X \cdot \sec^2 3X \cdot 3 + \sec X \tan X \cdot \tan 3X$$

c.) $g(x) = \sin(\cos^3(x^4))$

$$g'(x) = \cos(\cos^3(X^4)) \cdot 3\cos^2(X^4) \cdot -\sin(X^4) \cdot 4X^3$$

d.) $y = x^5 + 8^{-x^2}$

$$Y' = 5X^4 + 8^{-X^2} \cdot (-2X) \cdot \ln 8$$

e.) $y = \dfrac{4 - \ln x}{10 + \log_2(3x+7)}$

$$Y' = \frac{(10 + \log_2(3X+7)) \cdot \frac{-1}{X} - (4 - \ln X) \cdot \frac{1}{3X+7} \cdot 3 \cdot \frac{1}{\ln 2}}{(10 + \log_2(3X+7))^2}$$

f.) $y = x^{\ln x}$

$$\ln Y = \ln X^{\ln X} = \ln X \cdot \ln X = (\ln X)^2 \xrightarrow{D}$$

$$\frac{1}{Y} Y' = 2(\ln X) \cdot \frac{1}{X} \rightarrow Y' = X^{\ln X} \cdot 2\frac{\ln X}{X}$$

2.) Use $\lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$ to differentiate the function $f(x) = \dfrac{x+7}{3-x}$.

$$f'(x) = \lim_{h\to 0} \frac{\dfrac{(x+h)+7}{3-(x+h)} - \dfrac{x+7}{3-x}}{h/1}$$

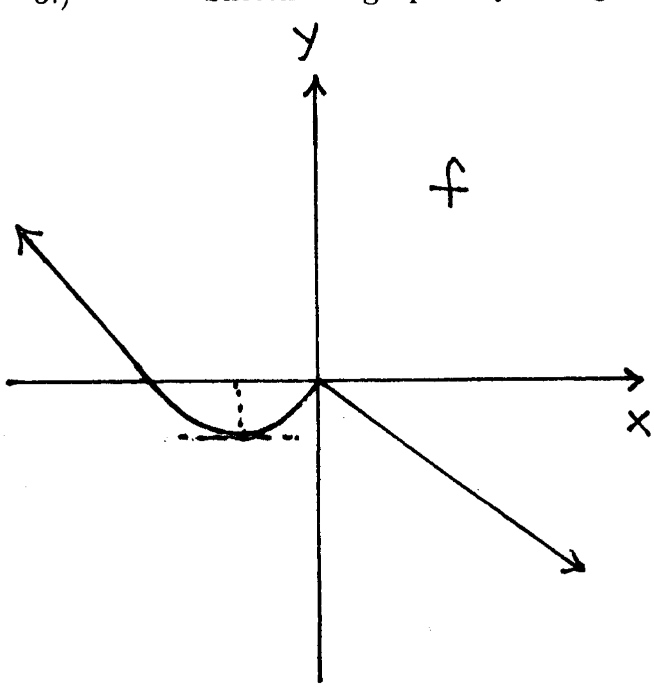
$$= \lim_{h\to 0} \frac{(x+h+7)(3-x) - (x+7)(3-x-h)}{(3-x-h)\ (3-x)} \cdot \frac{1}{h}$$

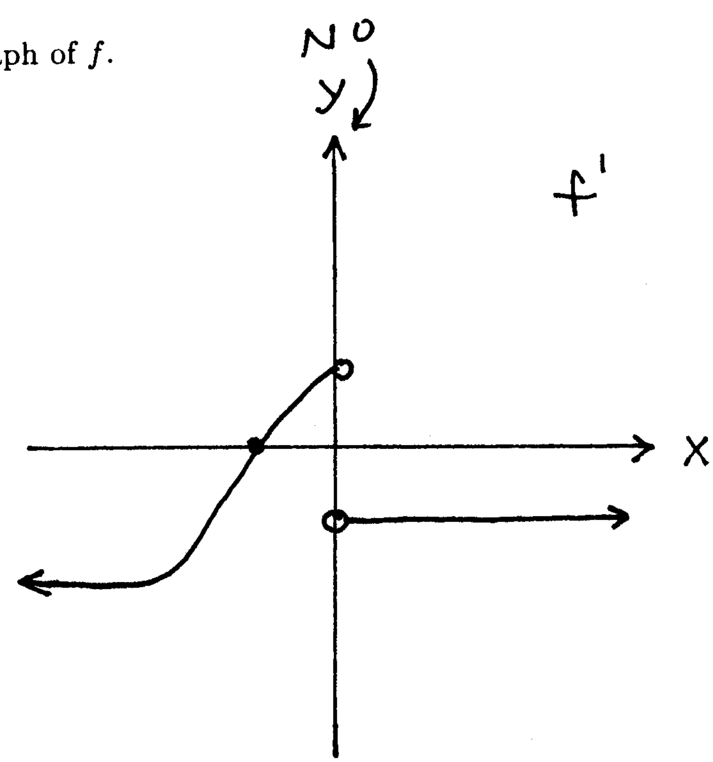
$$= \lim_{h\to 0} \frac{3x+3h+21-x^2-hx-7x-[3x-x^2-hx+21-7x-7h]}{(3-x-h)(3-x)\cdot h}$$

$$= \lim_{h\to 0} \frac{\cancel{3x}+3h+\cancel{21}-\cancel{x^2}-\cancel{hx}-\cancel{7x}-\cancel{3x}+\cancel{x^2}+\cancel{hx}-\cancel{21}+\cancel{7x}+7h}{(3-x-h)(3-x)\cdot h}$$

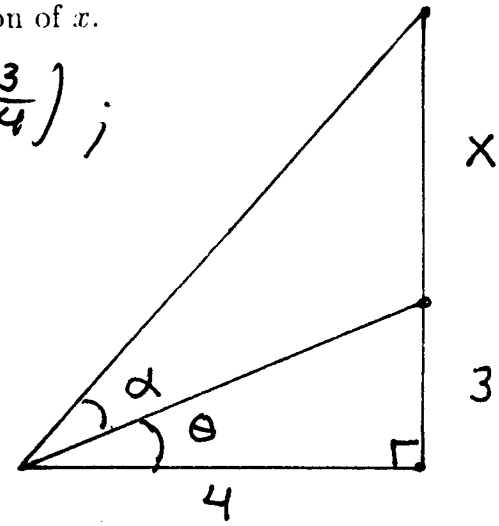
$$= \lim_{h\to 0} \frac{10\cancel{h}}{(3-x-h)(3-x)\cancel{h}} = \frac{10}{(3-x)^2}$$

3.) Sketch the graph of f' using the graph of f.





7.) Consider the given diagram. Write α as a function of x.



$$\tan\theta = \frac{3}{4} \rightarrow \theta = \arctan\left(\frac{3}{4}\right);$$

$$\tan(\alpha+\theta) = \frac{x+3}{4} \rightarrow$$

$$\alpha + \theta = \arctan\left(\frac{x+3}{4}\right) \rightarrow$$

$$\alpha = \arctan\left(\frac{x+3}{4}\right) - \theta \rightarrow$$

$$\alpha = \arctan\left(\frac{x+3}{4}\right) - \arctan\left(3/4\right)$$

8.) Let $f(x) = x + 5\arctan(1/x)$. Solve $f'(x) = 0$ for x.

$$f'(x) = 1 + 5 \cdot \frac{1}{1+\left(\frac{1}{x}\right)^2} \cdot \frac{-1}{x^2}$$

$$= 1 + \frac{-5}{x^2+1} = \frac{x^2+1}{x^2+1} - \frac{5}{x^2+1}$$

$$= \frac{x^2-4}{x^2+1} = \frac{(x-2)(x+2)}{x^2+1} = 0 \rightarrow$$

$$(x-2)(x+2) = 0 \longrightarrow x = 2,\ x = -2$$

9.) Differentiate the following function and SIMPLIFY your answer as much as possible : $f(x) = (x-3)\sqrt{6x-x^2} + 9\arcsin\left(\frac{x-3}{3}\right)$.

$$f'(x) = (x-3)\cdot\frac{1}{2}(6x-x^2)^{-1/2}\cdot(6-2x) + (1)\cdot\sqrt{6x-x^2} + 9\cdot\frac{1}{\sqrt{1-\left(\frac{x-3}{3}\right)^2}}\cdot\frac{1}{3}$$

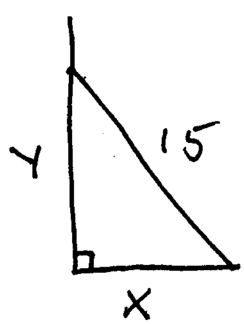
$$= \frac{(x-3)(3-x)}{\sqrt{6x-x^2}} + \frac{6x-x^2}{\sqrt{6x-x^2}} + \frac{3}{\sqrt{\frac{9}{9} - \frac{x^2-6x+9}{9}}}$$

$$= \frac{3x-x^2-9+3x+6x-x^2}{\sqrt{6x-x^2}} + \frac{3}{\frac{\sqrt{6x-x^2}}{3}}$$

$$= \frac{12x-2x^2-9}{\sqrt{6x-x^2}} + \frac{9}{\sqrt{6x-x^2}} = \frac{2(6x-x^2)}{\sqrt{6x-x^2}}$$

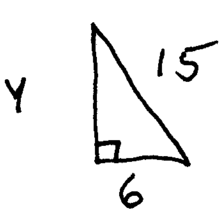
$$= 2\sqrt{6x-x^2}$$

8.) A 15-foot ladder is leaning against a wall. If the base of the ladder is pushed toward the wall at the rate of 2 ft./sec., at what rate is the top of the ladder moving up the wall when the base of the ladder is 6 ft. from the wall ?



assume $\frac{dx}{dt} = -2$ ft./sec.; find $\frac{dY}{dt}$ when $x = 6$ ft. :

$$x^2 + Y^2 = 15^2 \xrightarrow{D} 2x\cdot\frac{dx}{dt} + 2Y\frac{dY}{dt} = 0$$



$6^2 + Y^2 = 15^2 \longrightarrow Y = \sqrt{189} \approx 13.7$

$$(6)(-2) + \sqrt{189}\cdot\frac{dY}{dt} = 0 \longrightarrow$$

$$\frac{dY}{dt} = \frac{12}{\sqrt{189}} \approx 0.87 \text{ ft./sec.}$$

9.) Use differentials to estimate the value of $\sqrt{96}$.

Let $f(x) = \sqrt{x}$, $x: 100 \to 96$ so $\Delta x = -4$,

$f'(x) = \frac{1}{2\sqrt{x}}$; $\Delta f = f(96) - f(100) = \sqrt{96} - 10$,

$df = f'(100) \cdot \Delta x = \frac{1}{20} \cdot (-4) = \frac{-1}{5}$; assume $\Delta f \approx df \to$

$$\sqrt{96} - 10 \approx \frac{-1}{5} \longrightarrow \sqrt{96} \approx 9.8$$

10.) The radius and height of a cylinder are both equal to x so that the volume of the cylinder is given by $V = \pi x^3$. Assume that x is measured with an absolute percentage error of at most 3%. Use a differential to estimate the maximum absolute percentage error in computing the cylinder's volume.

assume $\frac{|\Delta x|}{x} \leq 3\%$; find $\frac{|\Delta V|}{V}$:

$V = \pi x^3 \longrightarrow V' = 3\pi x^2$, then

$$\frac{|\Delta V|}{V} \approx \frac{|dV|}{V} = \frac{|V' \cdot \Delta x|}{V} = \frac{3\cancel{\pi} x^2 \cdot |\Delta x|}{\cancel{\pi} x^3}$$

$$= 3 \cdot \frac{|\Delta x|}{x} \leq 3(3\%) = 9\%$$

The following EXTRA CREDIT PROBLEM is worth points. This problem is OPTIONAL.

1.) A beetle crawls along a thin rod on the x-axis from $x = 0$ in. to $x = 16$ in. at the rate of 3 in./min. The temperature of the rod at point x is $40 + 12\sqrt{x}$ degrees Fahrenheit (^{o} F). At what rate (^{o} F per min.) is the temperature of the rod under the beetle changing when the beetle is at $x = 9$ in. ?

$$\frac{dx}{dt} = 3 \text{ in./min}$$

$$T = 40 + 12\sqrt{x} \xrightarrow{D}$$

$$\frac{dT}{dt} = 12 \cdot \frac{1}{2} x^{-1/2} \cdot \frac{dx}{dt}$$

$$= \frac{6}{\sqrt{x}} \cdot \frac{dx}{dt} \quad \text{and } x = 9 \text{ in.} \longrightarrow$$

$$\frac{dT}{dt} = \frac{6}{3} \cdot 3 = 6 \text{ }^oF/\text{min}.$$