Happy Thanksg

11/23/

Final Review Part 1

Recall: Defn of limit $\lim_{x \to a} f(x) = L$

means* $\forall \varepsilon > 0, \exists \delta > 0$ s.t. $|x-a| < \delta \to |f(x) - L| < \varepsilon$

Prove $\lim_{x \to 1} \frac{2}{x+3} = \frac{1}{2}$

Study

1. go over MT #1 & 2
   - some prob. gonna be on test
2. practice midterm
3. Discussion sht
4. Hw

Proof: δ $\varepsilon > 0$ is given - find $\delta > 0$ s.t. $|x-1| < \delta \to \left| \underbrace{\frac{2}{x+3}}_{f(x)} - \frac{1}{2} \right| < \varepsilon$

start w/ $|f(x) - \frac{1}{2}| < \varepsilon$ + algebraically manipulate to obtain $|x-1| <$

$$\left| \frac{2}{x+3} - \frac{1}{2} \right| < \varepsilon \xrightarrow{\text{LCD}} \left| \frac{4-(x+3)}{x+3(2)} \right| < \varepsilon \longrightarrow \left| \frac{\overset{-(x-1)}{1-x}}{2(x+3)} \right| < \varepsilon$$

abs. value "kill" $\ominus$

$\frac{|x-1|}{2|x+3|} < \varepsilon$ want, remove

Assume $\delta \le 1$

a-δ, a+δ

0, a=1, 2

$0 < x < 2$

$3 < |x+3| < 5$ flip over: the sign also switc

$\boxed{\frac{1}{3} > \frac{1}{|x+3|} > \frac{1}{5}}$ cap/bound

$\frac{1}{2|x+3|} |x-1| < \varepsilon$

$\frac{1}{2 \cdot 3} |x-1| < \varepsilon$

$|x-1| < 6\varepsilon$

Choose $\delta = \min\{1, 6\varepsilon\}$

Hence when $|x-1| < \delta \to |f(x) - \frac{1}{2}| < \varepsilon$ ∎

Using algebra tricks to solve limits

$$\lim_{x \to 4} \frac{\sqrt{x}-2}{4-x} = \text{"}\frac{0}{0}\text{"}, \ \frac{\sqrt{x}+2}{\sqrt{x}+2} = \lim_{x \to 4} \frac{-1\,\overset{\text{reverse}}{\cancel{x-4}}}{\cancel{4-x}(\sqrt{x}+2)} = -\frac{1}{4}$$

③ For a limit to exist → need "KJS condit."

$\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x)$ if true then $\lim_{x \to a} f(x)$

Ex $\lim_{x \to 2} \frac{\sqrt{2x}\,(x-2)}{|x-2|}$ "$\frac{0}{0}$" Find limit

$\lim_{x \to 2^+} \frac{\sqrt{2x}\,(x-2)}{|x-2|} = \lim_{x \to 2^+} \frac{\sqrt{2x}\,\cancel{(x-2)}}{\cancel{x-2}} = 2$

2 # bigger, # smaller

Thus this is (+)

$\lim_{x \to 2^-} \frac{\sqrt{2x}\,(x-2)}{|x-2|} = \lim_{x \to 2^+} \frac{\sqrt{2x}\,(x-2)}{-(x-2)} = -2$

(−)

don't match ∴ limit DNE

④ 1st Applicatn of limit - Graph sketching

"division by zero"

Recall V.A: an x-value "a" s.t. $\lim_{x \to a^{\pm}} f(x) = \pm\infty$

H.A.: a y-value "L" s.t. $\lim_{x \to \pm\infty} f(x) = L$

look @ 1st midterm

Sketch $y = \frac{x+2}{x^2(x+1)} = \frac{x+2}{x^3+x^2}$

top = zero

x-int: $y = 0 \rightarrow x = -2$

y-int: $x = 0 \rightarrow$ DNE

V.A. look @ denom $x = 0, -1$

$\lim_{x \to 0^+} \frac{x+2}{x^2(x+1)} = \cancel{\frac{x+2}{x^3+x^3}} = +\infty$

$x > 0$ (+)

$x \geq 0$

$\frac{2}{0^+} = +\infty$

$\lim_{x \to 0^-} f(x) = \frac{2}{0^+} = +\infty$ } $x = 0$

$\lim_{x \to -1^+} \frac{x+2}{x^3+x^2} = \frac{x+2}{x^2(x+1)} = +\infty$

-0.5 (+) -0.5

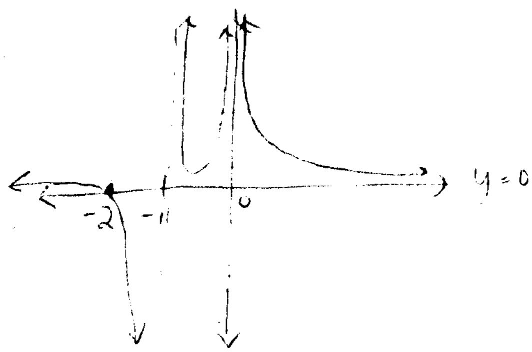
$\lim_{x \to -1^-} \frac{x+2}{x^2(x+1)} = -\infty$

-1.1 (+) (−) -1.1

} $x = -1$

H.A. $\lim\limits_{x \to +\infty} \dfrac{x+2}{x^3+x^2} \cdot \dfrac{1/x^3}{1/x^3} = \dfrac{\cancel{\frac{1}{x^2}}^{\,0} + \cancel{\frac{2}{x^3}}^{\,0}}{1 + \cancel{\frac{1}{x}}^{\,0}} = 0$

$\lim\limits_{x \to -\infty} f(x) = 0$ H.A. @ $y = 0$



⑤ Continuity (3 step defn)

f is cont. @ $x = a$ if:

1. $f(a)$ exists (existance condit.)
2. $\lim\limits_{x \to a} f(x)$ exists (kiss condit.)
3. $\lim\limits_{x \to a} f(x) = f(a)$ (match)

Ex. If f is conti @ $x = 2$

$$f(x) \begin{cases} x+1 & x > 2 \\ 2 & x = 2 \\ \dfrac{\sqrt{2x}\,(x-2)}{|x-2|} & x < 2 \end{cases}$$

1. $f(a) = 2$ exists
2. $\lim\limits_{x \to 2^+} x + 1 = 3$

$\lim\limits_{x \to 2^-} \dfrac{\sqrt{2x}\,(x-2)}{|x-2|} = -2$

} doesn't match ∴ $f(x)$ is discont. @ $x = 2$

6 IMVT *know thm!

✓ list
1. Define f
2. " m
3. est. f is conti. (state reasons)
4. Choose interval
5. Show m is b/t the interval - indicate m is b/t f(a) + f(b)

Look @ last midterm

Show $x^2 + 3x = \sqrt{x+1}$ are solvable
let $f(x) = x^2 + 3x - \sqrt{x+1} = 0$ ← let $m = 0$
$a = 0 \rightarrow f(a) < m$
$b = 3 \rightarrow f(b) > m$
interval $[0, 3]$
f is conti (sum of conti fn)
$f(a) < m < f(b)$
By IMVT....

extremely brief.