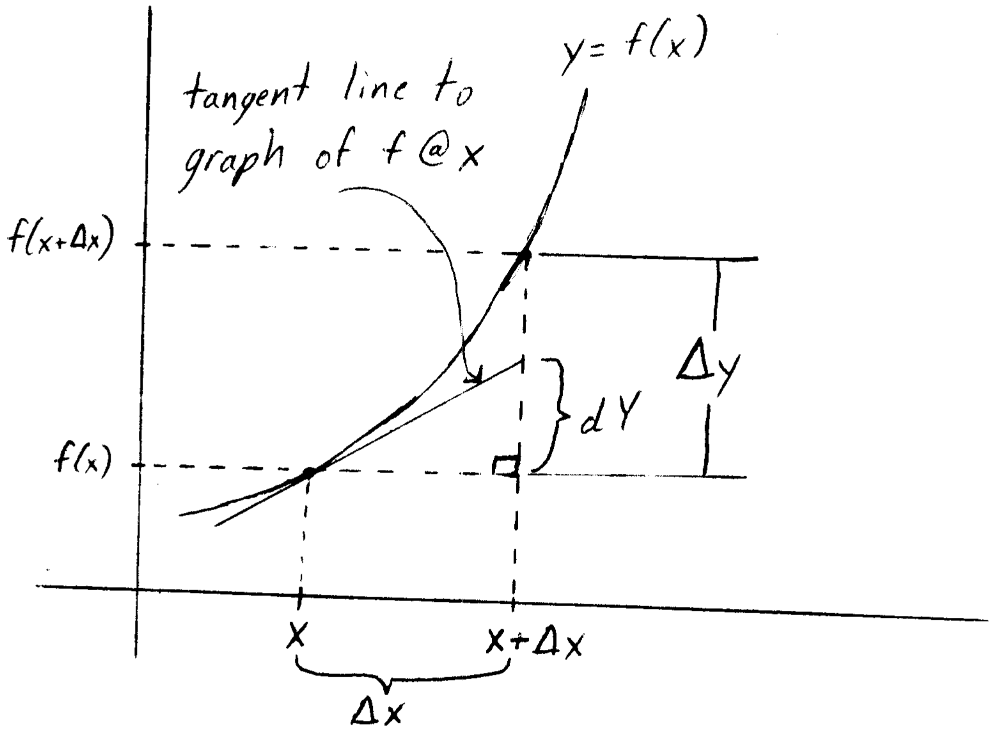
Math 21A
Vogler
The Differential



Define the <u>exact change</u> in f is

$$\boxed{\Delta y = \Delta f = f(x+\Delta x) - f(x)}$$

Assume x changes to $x+\Delta x$ ($x : x \to x+\Delta x$)

Note slope $= \frac{\text{rise}}{\text{run}} \Rightarrow f'(x) = \frac{dy}{\Delta x} \Rightarrow dy = f'(x)\Delta x$

Define the <u>differential</u> (approximate change) of f is

$$\boxed{dy = df = f'(x)\,\Delta x}$$

With the differential, we can approximate/simplify fns by the following eqn for a line

$$\boxed{f(x+\Delta x) \approx f(x) + df = f(x) + f'(x)\,\Delta x}$$

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More examples using differentials

Example 1: For small h, show that
$\sqrt{4+3h^2} \approx 2 + \frac{3}{4}h^2$ using differentials.

Soln Let $f(x) = \sqrt{x}$ & assume that $x: 4 \to 4+3h^2$

$\Rightarrow \Delta x = 3h^2$ & $f'(x) = \frac{1}{2\sqrt{x}}$. Since Δx is small (b/c h is small)

$\Delta f \approx df \Rightarrow f(4+3h^2) - f(4) \approx f'(4) \cdot \Delta x$

$\Rightarrow \sqrt{4+3h^2} - \sqrt{4} \approx \frac{1}{2\sqrt{4}} \cdot 3h^2 \Rightarrow \sqrt{4+3h^2} - 2 \approx \frac{3}{4}h^2$

$\Rightarrow \sqrt{4+3h^2} \approx 2 + \frac{3}{4}h^2$

Example 2: If the radius of a circle is measured w/ an absolute percentage error of @ most 3%, use differentials to estimate the maximum absolute percentage error in computing the circle's

a) circumference b) area

Solution Assume that $\frac{|\Delta r|}{r} \le 3\%$

a) $C = 2\pi r \Rightarrow C' = 2\pi$, find $\frac{|\Delta C|}{C}$:

$$\frac{|\Delta C|}{C} \approx \frac{|dC|}{C} = \frac{|C'\Delta r|}{C} = \frac{|2\pi \cdot \Delta r|}{2\pi r} = \frac{|\Delta r|}{r} \le 3\%$$

b) $A = \pi r^2 \Rightarrow A' = 2\pi r$, find $\frac{|\Delta A|}{A}$

$$\frac{|\Delta A|}{A} \approx \frac{|dA|}{A} = \frac{|A'\Delta r|}{A} = \frac{|2\pi r \Delta r|}{\pi r^2} = 2\frac{|\Delta r|}{r} \le 2(3\%) = 6\%$$