Sec. 3.5

1.) $y = -10x + 3\cos x \xrightarrow{D} y' = -10 - 3\sin x$

2.) $y = \frac{3}{x} + 5\sin x \xrightarrow{D} y' = \frac{-3}{x^2} + 5\cos x$

9.) $y = (\sec x + \tan x)(\sec x - \tan x) \xrightarrow{D}$

$y' = (\sec x + \tan x)(\sec x \tan x - \sec^2 x)$

$\quad + (\sec x \tan x + \sec^2 x)(\sec x - \tan x)$

12.) $y = \frac{\cos x}{1 + \sin x} \xrightarrow{D} y' = \frac{(1+\sin x)(-\sin x) - \cos x(\cos x)}{(1+\sin x)^2}$

13.) $y = \frac{4}{\cos x} + \frac{1}{\tan x} = 4\sec x + \cot x \xrightarrow{D}$

$y' = 4\sec x \tan x - \csc^2 x$

20.) $s = t^2 - \sec t + 5e^t \xrightarrow{D}$

$s' = 2t - \sec t \tan t + 5e^t$

24.) $r = \theta \sin\theta + \cos\theta \xrightarrow{D}$

$r' = \theta \cdot \cos\theta + (1) \cdot \sin\theta - \sin\theta$

34.) b.) $y = 9\cos x \longrightarrow$

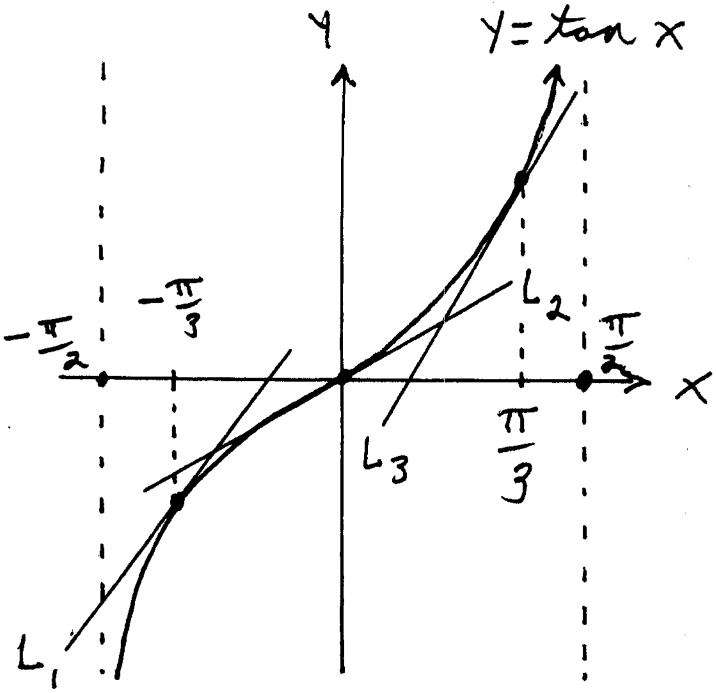
$y' = -9\sin x \longrightarrow$

$y'' = -9\cos x \longrightarrow$

$y''' = 9\sin x \longrightarrow$

$y^{(4)} = 9\cos x$

36.)



$Y' = \sec^2 x$

a.) $X = {}^{-}\pi/3 \rightarrow$ slope

$m = Y' = \sec^2\left(-\frac{\pi}{3}\right)$

$= \dfrac{1}{\cos^2(-\pi/3)}$

$= \dfrac{1}{(1/2)^2} = 4$;

$X = {}^{-}\frac{\pi}{3} \rightarrow Y = \tan\left(-\frac{\pi}{3}\right) = -\sqrt{3}/2 \div 1/2 = -\sqrt{3}$;

tangent line is

$L_1: Y - (-\sqrt{3}) = 4\left(X - {}^{-}\pi/3\right) \longrightarrow L_1: Y = 4X + \frac{4\pi}{3} - \sqrt{3}$

b.) $X = 0 \rightarrow$ slope $m = Y' = \sec^2(0) = 1$;

$X = 0 \rightarrow Y = \tan 0 = 0$; tangent line is

$L_2: Y - 0 = 1\cdot(X - 0) \rightarrow L_2: Y = X$

c.) $X = \frac{\pi}{3} \rightarrow$ slope $m = Y' = \sec^2\left(\frac{\pi}{3}\right) = 4$;

$X = \pi/3 \longrightarrow Y = \tan(\pi/3) = \sqrt{3}$; tangent

line is $L_3: Y - \sqrt{3} = 4(X - \pi/3) \longrightarrow$

$L_3: Y = 4X + \sqrt{3} - \frac{4\pi}{3}$

44.) $Y = -X \longrightarrow Y' = -1$; $Y = \cot x \longrightarrow$

$Y' = -\csc^2 x = \dfrac{-1}{\sin^2 x}$; if $\dfrac{-1}{\sin^2 x} = -1$,

then $\sin^2 x = 1 \longrightarrow$ $(0 < X < \pi)$

$$\left.\begin{array}{l}\sin x = 1\\ \text{or } \sin X = -1\end{array}\right\} \quad X = \pi/2$$

$= \sqrt{1+\cos(\pi\cdot(-2))} = \sqrt{1+1} = \sqrt{2}$

52.) $\lim_{x\to 0} \sin\left(\frac{\pi + \tan x}{\tan x - 2\sec x}\right)$

$= \sin\left(\frac{\pi + \tan 0}{\tan 0 - 2\sec 0}\right) = \sin\left(\frac{\pi + 0}{0 - 2(1)}\right)$

$= \sin\left(-\frac{\pi}{2}\right) = -1$

54.) $\lim_{\theta\to 0} \cos\left(\frac{\pi\theta}{\sin\theta}\right) = \lim_{\theta\to 0} \cos\left(\pi\cdot\frac{1}{\frac{\sin\theta}{\theta}}\right)$

$= \cos(\pi\cdot 1) = -1$

58.) $g(x) = \begin{cases} x+b, & \text{if } x<0 \\ \cos x, & \text{if } x\geq 0 \end{cases}$;

a.) make g continuous at $x=0$:

i.) $g(0) = \cos 0 = 1$

ii.) $\lim_{x\to 0} g(x)$ must be 1 ! :

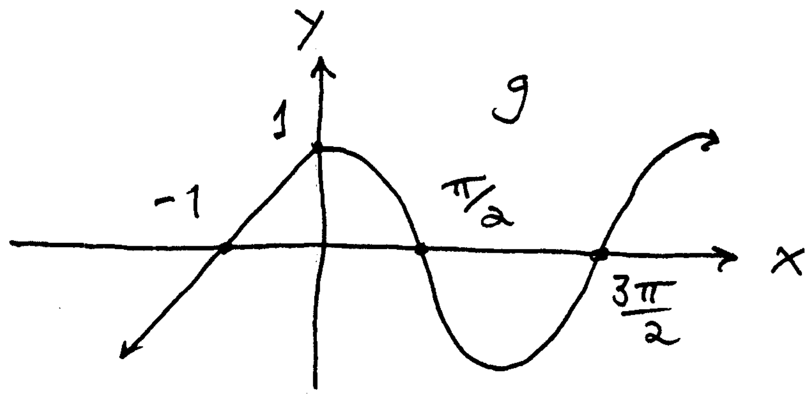
$\lim_{x\to 0^+} g(x) = \lim_{x\to 0^+} \cos x = \cos 0 = 1$;

$\lim_{x\to 0^-} g(x) = \lim_{x\to 0^-} (x+b) = 0+b = b$, so

$\boxed{b=1}$ and $g(x) = \begin{cases} x+1, & \text{if } x<0 \\ \cos x, & \text{if } x\geq 0 \end{cases}$;

b.) Is g differentiable at $x=0$?

$g'(0) = \lim_{h\to 0} \frac{g(0+h) - g(0)}{h}$



$$= \lim_{h \to 0} \frac{g(h) - 1}{h} \quad ;$$

$$\lim_{h \to 0^+} \frac{g(h) - 1}{h} = \lim_{h \to 0^+} \frac{\cos h - 1}{h}$$

$$\overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{h \to 0} \frac{\cos h - 1}{h} \cdot \frac{\cos h + 1}{\cos h + 1}$$

$$= \lim_{h \to 0} \frac{\cos^2 h - 1}{h(\cos h + 1)} = \lim_{h \to 0} \frac{-\sin^2 h}{h(\cos h + 1)}$$

$$= \lim_{h \to 0} -\frac{\sin h}{h} \cdot \frac{\sin h}{\cos h + 1}$$

$$= -(1) \cdot \frac{0}{1+1} = \boxed{0} \quad ;$$

$$\lim_{h \to 0^-} \frac{g(h) - 1}{h} = \lim_{h \to 0^-} \frac{(h + \cancel{1}) - \cancel{1}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}}{\cancel{h}} = \lim_{h \to 0} 1 = \boxed{1} \quad ;$$

so $\boxed{g'(0) \quad \text{DNE}}$!

59.) $\frac{d}{dx}(\cos x) = -\sin x$,

$\frac{d^2}{dx^2}(\cos x) = -\cos x$,

→ $\frac{d^3}{dx^3}(\cos x) = \sin x$,

$\frac{d^4}{dx^4}(\cos x) = \cos x$; thus

$\frac{d^8}{dx^8}(\cos x) = \cos x$,

$\frac{d^{12}}{dx^{12}}(\cos x) = \cos x$, ...

$\frac{d^{996}}{dx^{996}}(\cos x) = \frac{d^{4 \cdot 249}}{dx^{4 \cdot 249}}(\cos x) = \cos x$, ...

$\frac{d^{999}}{dx^{999}}(\cos x) = \sin x$