Section 3.7

1.) $X^2Y + XY^2 = 6 \xrightarrow{D}$

$X^2 \cdot Y' + 2X \cdot Y + X \cdot 2YY' + (1) \cdot Y^2 = 0 \rightarrow$

$Y'(X^2 + 2XY) = -2XY - Y^2 \rightarrow$

$$Y' = \frac{-2XY - Y^2}{X^2 + 2XY}$$

4.) $X^3 - XY + Y^3 = 1 \xrightarrow{D}$

$3X^2 - (X \cdot Y' + (1) \cdot Y) + 3Y^2 \cdot Y' = 0 \rightarrow$

$3X^2 - XY' - Y + 3Y^2Y' = 0 \rightarrow$

$Y'(3Y^2 - X) = Y - 3X^2 \rightarrow$

$$Y' = \frac{Y - 3X^2}{3Y^2 - X}$$

6.) $(3XY + 7)^2 = 6Y \xrightarrow{D}$

$\cancel{2}(3XY + 7) \cdot (3X \cdot Y' + 3 \cdot Y) = \overset{3}{\cancel{6}}Y' \rightarrow$

$9X^2YY' + 21XY' + 9XY^2 + 21Y = 3Y' \rightarrow$

$9X^2YY' + 21XY' - 3Y' = -9XY^2 - 21Y \rightarrow$

$\cancel{3}Y'(3X^2Y + 7X - 1) = \cancel{3}(-3XY^2 - 7Y) \rightarrow$

$$Y' = \frac{-3XY^2 - 7Y}{3X^2Y + 7X - 1}$$

9.) $X = \tan Y \xrightarrow{D}$

$1 = \sec^2 Y \cdot Y' \rightarrow \quad Y' = \frac{1}{\sec^2 Y} \rightarrow$

$Y' = \cos^2 Y$

Ex.) $X + \sin Y = XY \xrightarrow{D}$

$1 + \cos Y \cdot Y' = X \cdot Y' + (1) \cdot Y \rightarrow$

$$\cos Y \cdot Y' - xY' = Y - 1 \longrightarrow$$

$$Y'(\cos Y - x) = Y - 1 \longrightarrow$$

$$Y' = \frac{Y-1}{\cos Y - x}$$

16.) $$e^{x^2 Y} = 2x + 2Y \xrightarrow{D}$$

$$e^{x^2Y} \cdot (x^2 Y' + 2x \cdot Y) = 2 + 2Y' \longrightarrow$$

$$x^2 e^{x^2Y} \cdot Y' + 2x e^{x^2Y} \cdot Y = 2 + 2Y' \longrightarrow$$

$$x^2 e^{x^2Y} \cdot Y' - 2Y' = 2 - 2xYe^{x^2Y} \longrightarrow$$

$$Y'(x^2 e^{x^2Y} - 2) = 2 - 2xYe^{x^2Y} \longrightarrow$$

$$Y' = \frac{2 - 2xYe^{x^2Y}}{x^2 e^{x^2Y} - 2}$$

21.) $$x^2 + Y^2 = 1 \xrightarrow{D}$$

$$2x + 2YY' = 0 \rightarrow 2YY' = -2x \longrightarrow$$

$$Y' = \frac{-2x}{2Y} \longrightarrow \boxed{Y' = \frac{-x}{Y}} \; ; \xrightarrow{D}$$

$$Y'' = \frac{Y \cdot (-1) - (-x) \cdot Y'}{Y^2}$$

$$= \frac{-Y + x \cdot Y'}{Y^2} = \frac{-Y + x \cdot \left(\frac{-x}{Y}\right)}{Y^2} \cdot \frac{Y}{Y}$$

$$= \frac{-Y^2 - x^2}{Y^3} = \frac{-(x^2 + Y^2)}{Y^3} = \frac{-1}{Y^3} \longrightarrow$$

$$\boxed{Y'' = \frac{-1}{Y^3}}$$

24.) $$xY + Y^2 = 1 \xrightarrow{D}$$

$$xy' + (1)y + 2yy' = 0 \rightarrow$$

$$y'(x+2y) = -y \rightarrow \boxed{y' = \frac{-y}{x+2y}} \; ; \; \xrightarrow{D}$$

$$y'' = \frac{(x+2y)\cdot -y' - (-y)\cdot(1+2y')}{(x+2y)^2}$$

$$= \frac{-(x+2y)\cdot\frac{-y}{x+2y} + y + 2y\cdot\frac{-y}{x+2y}}{(x+2y)^2} \cdot \frac{x+2y}{x+2y}$$

$$= \frac{(x+2y)\cdot y + y\cdot(x+2y) - 2y^2}{(x+2y)^3}$$

$$= \frac{xy + 2y^2 + xy + \cancel{2y^2} - \cancel{2y^2}}{(x+2y)3}$$

$$= \frac{2xy + 2y^2}{(x+2y)^3} \rightarrow \boxed{y'' = \frac{2xy+2y^2}{(x+2y)^3}}$$

39.) $y = 2\sin(\pi x - y)$, if $x = 1$, $y = 0$ then

$0 = 2\sin(\pi) = 2(0) = 0$; $\xrightarrow{D}$

$$y' = 2\cos(\pi x - y)\cdot(\pi - y')$$

$$= 2\pi\cos(\pi x - y) - 2\cos(\pi x - y)\cdot y' \rightarrow$$

$$y' + 2\cos(\pi x - y)\cdot y' = 2\pi\cos(\pi x - y) \rightarrow$$

$$y'(1 + 2\cos(\pi x - y)) = 2\pi\cos(\pi x - y) \rightarrow$$

$$y' = \frac{2\pi\cos(\pi x - y)}{1 + 2\cos(\pi x - y)} \; ;$$

a.) $x = 1$, $y = 0$ so slope $y' = \frac{2\pi\cos\pi}{1 + 2\cos\pi}$

$= \frac{2\pi(-1)}{1+2(-1)} = 2\pi$; tangent line is

$$y - 0 = 2\pi(x - 1) \rightarrow y = 2\pi x - 2\pi$$

b.) $x=1$, $y=0$ so $y'=2\pi$ and slope is $m=\frac{-1}{2\pi}$; $\perp$ line is

$y-0=\frac{-1}{2\pi}(x-1) \rightarrow y=\frac{-1}{2\pi}x+\frac{1}{2\pi}$.

41.) $x^2+xy+y^2=7$; if $y=0$ (crosses x-axis) $\rightarrow x^2=7 \rightarrow x=\sqrt{7}$, $x=-\sqrt{7}$;

$\xrightarrow{D} 2x+xy'+(1)y+2yy'=0 \rightarrow$

$y'(x+2y)=-2x-y \rightarrow y'=\frac{-2x-y}{x+2y}$;

slope at $(\sqrt{7},0)$ is $y'=\frac{-2\sqrt{7}}{\sqrt{7}}=-2$;

slope at $(-\sqrt{7},0)$ is $y'=\frac{2\sqrt{7}}{-\sqrt{7}}=-2$.

44.) $2y^2-xy^2=x^3 \xrightarrow{D}$

$4yy'-(x\cdot 2yy'+(1)y^2)=3x^2 \rightarrow$

$4yy'-2xyy'-y^2=3x^2 \rightarrow$

$y'(4y-2xy)=3x^2+y^2 \rightarrow$

$y'=\frac{3x^2+y^2}{4y-2xy}$

a.) $x=1$, $y=1 \rightarrow$ slope $y'=\frac{3+1}{4-2}=2$

so tangent line is

$y-1=2(x-1) \rightarrow y=2x-1$

b.) $x=1$, $y=1 \rightarrow y'=2$ so slope $m=\frac{-1}{2} \rightarrow \perp$ line is

$y-1=\frac{-1}{2}(x-1)=\frac{-1}{2}x+\frac{1}{2} \rightarrow y=\frac{-1}{2}x+\frac{3}{2}$

47.) $\boxed{X^2 + 2XY - 3Y^2 = 0} \xrightarrow{D}$

$2X + 2X \cdot Y' + 2 \cdot Y - 6YY' = 0 \rightarrow$

$Y'(2X - 6Y) = -2X - 2Y \rightarrow$

$Y' = \dfrac{-2X - 2Y}{2X - 6Y} = \dfrac{\cancel{2}(-X - Y)}{\cancel{2}(X - 3Y)} = \dfrac{X + Y}{3Y - 1}$;

$X = 1,\ Y = 1 \rightarrow Y' = \frac{2}{2} = 1$ so slope $m = -1$ and $\perp$ line is

$Y - 1 = -1(X - 1) = -X + 1 \rightarrow \boxed{Y = -X + 2}$;

find pts. of intersection:

$X^2 + 2X(-X + 2) - 3(-X + 2)^2 = 0 \rightarrow$

$X^2 - 2X^2 + 4X - 3(X^2 - 4X + 4) = 0 \rightarrow$

$-X^2 + 4X - 3X^2 + 12X - 12 = 0 \rightarrow$

$-4X^2 + 16X - 12 = 0 \rightarrow$

$-4(X^2 - 4X + 3) = 0 \rightarrow$

$-4(X - 3)(X - 1) = 0 \rightarrow X = 1,\ Y = 1$

or $X = 3,\ Y = -1$

53.) $XY^3 + X^2Y = 6 \xrightarrow{D}$ (Y is function, X is variable)

$X \cdot 3Y^2Y' + (1) \cdot Y^3 + X^2 \cdot Y' + 2X \cdot Y = 0 \rightarrow$

$Y'(3XY^2 + X^2) = -2XY - Y^3 \rightarrow$

$Y' = \boxed{\dfrac{dY}{dX} = \dfrac{-2XY - Y^3}{3XY^2 + X^2}}$;

$XY^3 + X^2Y = 6 \xrightarrow{D}$ (X is function, Y is variable)

$X \cdot 3Y^2 + X' \cdot Y^3 + X^2 \cdot (1) + 2XX' \cdot Y = 0 \rightarrow$

$$x'(y^3 + 2xy) = -3xy^2 - x^2 \rightarrow$$

$$x' = \boxed{\frac{dx}{dy} = \frac{-3xy^2 - x^2}{2xy + y^3}} \; ;$$

it follows that

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} .$$