

Section 3.8

$$11.) \quad y = \ln 3x \quad \xrightarrow{D} \quad y' = \frac{1}{3x} \cdot 3 = \frac{1}{x}$$

$$16.) \quad y = \ln\left(\frac{10}{x}\right) = \ln 10 - \ln x \quad \xrightarrow{D}$$

$$y' = -\frac{1}{x}$$

$$19.) \quad y = \ln x^3 = 3 \cdot \ln x \quad \xrightarrow{D}$$

$$y' = 3 \cdot \frac{1}{x}$$

$$20.) \quad y = (\ln x)^3 \quad \xrightarrow{D} \quad y' = 3(\ln x)^2 \cdot \frac{1}{x}$$

$$22.) \quad y = t (\ln t)^{1/2} \quad \xrightarrow{D}$$

$$y' = t \cdot \frac{1}{2} (\ln t)^{-1/2} \cdot \left(\frac{1}{t}\right) + (1) \cdot (\ln t)^{1/2}$$

$$25.) \quad y = \frac{\ln t}{t} \quad \xrightarrow{D} \quad y' = \frac{t \cdot \left(\frac{1}{t}\right) - \ln t \cdot (1)}{t^2}$$

$$28.) \quad y = \frac{x \ln x}{1 + \ln x} \quad \xrightarrow{D}$$

$$y' = \frac{(1 + \ln x) \cdot \left(x \cdot \frac{1}{x} + (1) \ln x\right) - x \ln x \cdot \left(\frac{1}{x}\right)}{(1 + \ln x)^2}$$

$$30.) \quad y = \ln(\ln(\ln x)) \quad \xrightarrow{D}$$

$$y' = \frac{1}{\ln(\ln x)} \cdot \frac{1}{\ln x} \cdot \frac{1}{x}$$

$$32.) \quad y = \ln(\sec \theta + \tan \theta) \quad \xrightarrow{D}$$

$$y' = \frac{1}{\sec \theta + \tan \theta} \cdot (\sec \theta \tan \theta + \sec^2 \theta)$$

$$37.) \quad y = \ln(\sec(\ln \theta)) \quad \xrightarrow{D}$$

$$Y' = \frac{1}{\sec(\ln \theta)} \cdot \sec(\ln \theta) \tan(\ln \theta) \cdot \frac{1}{\theta}$$

$$39.) Y = \ln \frac{(x^2+1)^5}{(1-x)^{1/2}} = \ln(x^2+1)^5 - \ln(1-x)^{1/2}$$

$$= 5 \ln(x^2+1) - \frac{1}{2} \ln(1-x) \xrightarrow{D}$$

$$Y' = 5 \cdot \frac{1}{x^2+1} \cdot 2x - \frac{1}{2} \cdot \frac{1}{1-x} \cdot (-1)$$

$$40.) Y = \ln \left(\frac{(x+1)^5}{(x+2)^{20}} \right)^{1/2} = \ln \frac{(x+1)^{5/2}}{(x+2)^{10}}$$

$$= \ln(x+1)^{5/2} - \ln(x+2)^{10} = \frac{5}{2} \ln(x+1) - 10 \ln(x+2) \xrightarrow{D}$$

$$Y' = \frac{5}{2} \cdot \frac{1}{x+1} - 10 \cdot \frac{1}{x+2}$$

$$51.) Y = \frac{x(x^2+1)^{1/2}}{(x+1)^{2/3}} \rightarrow \ln Y = \ln \frac{x \cdot (x^2+1)^{1/2}}{(x+1)^{2/3}}$$

$$\rightarrow \ln Y = \ln x + \frac{1}{2} \ln(x^2+1) - \frac{2}{3} \ln(x+1) \xrightarrow{D}$$

$$\frac{1}{Y} Y' = \frac{1}{x} + \frac{1}{2} \cdot \frac{2x}{x^2+1} - \frac{2}{3} \cdot \frac{1}{x+1} \rightarrow$$

$$Y' = \frac{x(x^2+1)^{1/2}}{(x+1)^{2/3}} \cdot \left[\frac{1}{x} + \frac{x}{x^2+1} - \frac{2}{3} \cdot \frac{1}{x+1} \right]$$

$$54.) Y = \left(\frac{x(x+1)(x-2)}{(x^2+1)(2x+3)} \right)^{1/3} \rightarrow$$

$$\ln Y = \ln \left(\frac{x^{1/3} \cdot (x+1)^{1/3} \cdot (x-2)^{1/3}}{(x^2+1)^{1/3} (2x+3)^{1/3}} \right) \rightarrow$$

$$\ln Y = \frac{1}{3} \ln x + \frac{1}{3} \ln(x+1) + \frac{1}{3} \ln(x-2)$$

$$- \frac{1}{3} \ln(x^2+1) - \frac{1}{3} \ln(2x+3) \xrightarrow{D}$$

$$\frac{1}{Y} Y' = \frac{1}{3} \cdot \frac{1}{X} + \frac{1}{3} \cdot \frac{1}{X+1} + \frac{1}{3} \cdot \frac{1}{X-2}$$

$$- \frac{1}{3} \cdot \frac{2X}{X^2+1} - \frac{1}{3} \cdot \frac{2}{2X+3} \rightarrow$$

$$Y' = \left(\frac{X(X+1)(X-2)}{(X^2+1)(2X+3)} \right)^{1/3} \left[\frac{1}{3X} + \frac{1}{3(X+1)} + \frac{1}{3(X-2)} \right.$$

$$\left. - \frac{2X}{3(X^2+1)} - \frac{2}{3(2X+3)} \right]$$

59.) $Y = \ln \left(\frac{e^\theta}{1+e^\theta} \right) = \ln e^\theta - \ln(1+e^\theta)$

$$= \theta - \ln(1+e^\theta) \xrightarrow{D}$$

$$Y' = 1 - \frac{1}{1+e^\theta} \cdot e^\theta$$

61.) $Y = e^{(\cos t + \ln t)} \xrightarrow{D}$

$$Y' = e^{(\cos t + \ln t)} \cdot (-\sin t + \frac{1}{t})$$

65.) $X^Y = Y^X \rightarrow \ln X^Y = \ln Y^X \rightarrow$

$$Y \cdot \ln X = X \cdot \ln Y \xrightarrow{D}$$

$$Y \cdot \frac{1}{X} + Y' \cdot \ln X = X \cdot \frac{1}{Y} Y' + (1) \cdot \ln Y \rightarrow$$

$$Y' \cdot \ln X - \frac{X}{Y} Y' = \ln Y - \frac{Y}{X} \rightarrow$$

$$Y' \left(\ln X - \frac{X}{Y} \right) = \ln Y - \frac{Y}{X} \rightarrow$$

$$Y' = \frac{\ln Y - \frac{Y}{X}}{\ln X - \frac{X}{Y}}$$

$$66.) \tan y = e^x + \ln x \xrightarrow{D}$$

$$\sec^2 y \cdot y' = e^x + \frac{1}{x} \rightarrow$$

$$y' = \frac{e^x + \frac{1}{x}}{\sec^2 y}$$

$$68.) y = 3^{-x} \xrightarrow{D} y' = 3^{-x} \cdot (-1) \cdot \ln 3$$

$$71.) y = x^\pi \xrightarrow{D} y' = \pi x^{\pi-1}$$

$$74.) y = \log_3 (1 + \theta \ln 3) \xrightarrow{D}$$

$$y' = \frac{1}{1 + \theta \ln 3} \cdot \cancel{\ln 3} \cdot \frac{1}{\cancel{\ln 3}} = \frac{1}{1 + \theta \ln 3}$$

$$82.) y = \log_7 \left(\frac{\sin \theta \cdot \cos 2\theta}{e^\theta \cdot 2^\theta} \right)$$

$$= \log_7 (\sin \theta) + \log_7 (\cos 2\theta) - \log_7 e^\theta - \log_7 2^\theta$$

$$= \log_7 (\sin \theta) + \log_7 (\cos 2\theta) - \theta \cdot \log_7 e - \theta \log_7 2$$

$$\xrightarrow{D} y' = \frac{1}{\sin \theta} \cdot \cos 2\theta \cdot \frac{1}{\ln 7} + \frac{1}{\cos 2\theta} \cdot (-\sin 2\theta) \cdot \frac{1}{\ln 7}$$

$$- \log_7 e - \log_7 2$$

$$85.) y = 3^{\log_2 t} \xrightarrow{D} y' = 3^{\log_2 t} \cdot \ln 3 \cdot \frac{1}{2} \cdot \frac{1}{\ln 2}$$

$$86.) y = 3 \log_8 (\log_2 t) \xrightarrow{D}$$

$$y' = 3 \cdot \frac{1}{\log_2 t} \cdot \frac{1}{\ln 8} \cdot \frac{1}{t} \cdot \frac{1}{\ln 2}$$

$$90.) \quad y = x^{x+1} \rightarrow \ln y = \ln x^{x+1} = (x+1) \ln x \xrightarrow{D}$$

$$\frac{1}{y} \cdot y' = (x+1) \cdot \frac{1}{x} + (1) \cdot \ln x \rightarrow$$

$$y' = x^{x+1} \cdot \left(\frac{x+1}{x} + \ln x \right)$$

$$92.) \quad y = t^{\sqrt{t}} \rightarrow \ln y = \ln t^{\sqrt{t}} = \sqrt{t} \cdot \ln t \xrightarrow{D}$$

$$\frac{1}{y} y' = \sqrt{t} \cdot \frac{1}{t} + \frac{1}{2} t^{-1/2} \cdot \ln t \rightarrow$$

$$y' = t^{\sqrt{t}} \cdot \left(\frac{1}{\sqrt{t}} + \frac{1}{2} t^{-1/2} \cdot \ln t \right)$$

$$93.) \quad y = (\sin x)^x \rightarrow \ln y = \ln (\sin x)^x \rightarrow$$

$$\ln y = x \cdot \ln (\sin x) \xrightarrow{D}$$

$$\frac{1}{y} y' = x \cdot \frac{1}{\sin x} \cdot \cos x + (1) \cdot \ln (\sin x) \rightarrow$$

$$y' = (\sin x)^x \cdot [x \cot x + \ln (\sin x)]$$

$$96.) \quad y = (\ln x)^{\ln x} \rightarrow \ln y = \ln (\ln x)^{\ln x} \rightarrow$$

$$\ln y = \ln x \cdot \ln (\ln x) \xrightarrow{D}$$

$$\frac{1}{y} y' = \cancel{\ln x} \cdot \frac{1}{\cancel{\ln x}} \cdot \frac{1}{x} + \frac{1}{x} \cdot \ln (\ln x) \rightarrow$$

$$y' = (\ln x)^{\ln x} \cdot \left[\frac{1}{x} + \frac{\ln (\ln x)}{x} \right]$$