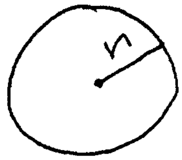
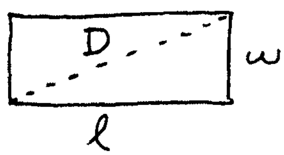
Section 3.10

20.) 

assume $\frac{dr}{dt} = 0.01$ cm./min.; find $\frac{dA}{dt}$ when $r = 50$ cm.:

$$A = \pi r^2 \xrightarrow{D} \frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt} = 2\pi(50)(0.01)$$

$$\rightarrow \frac{dA}{dt} = \pi \text{ cm.}^2/\text{min.}$$

21.) 

assume $\frac{dl}{dt} = -2$ cm./sec., $\frac{dw}{dt} = 2$ cm./sec.; when $l = 12$ cm., $w = 5$ cm.:

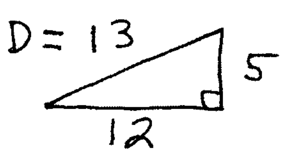
a.) Find $\frac{dA}{dt}$: $A = lw \xrightarrow{D}$

$$\frac{dA}{dt} = l \cdot \frac{dw}{dt} + \frac{dl}{dt} \cdot w = (12)(2) + (-2)(5) = 14 \ \frac{\text{cm.}^2}{\text{sec.}}$$

b.) Find $\frac{dP}{dt}$: $P = 2l + 2w \xrightarrow{D}$

$$\frac{dP}{dt} = 2 \cdot \frac{dl}{dt} + 2 \cdot \frac{dw}{dt} = 2(-2) + 2(2) = 0 \text{ cm./sec.}$$

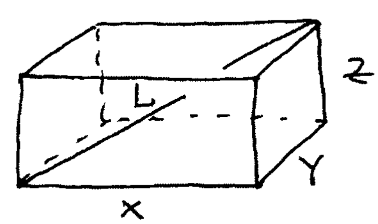
c.) Find $\frac{dD}{dt}$: $D^2 = l^2 + w^2 \xrightarrow{D}$



$$\cancel{2}D \cdot \frac{dD}{dt} = \cancel{2}l \cdot \frac{dl}{dt} + \cancel{2}w \cdot \frac{dw}{dt} \rightarrow$$

$$(13) \cdot \frac{dD}{dt} = (12)(-2) + (5)(2) \rightarrow$$

$$\frac{dD}{dt} = \frac{-14}{13} \text{ cm./sec.}$$

22.) assume 

$\frac{dx}{dt} = 1 \ \frac{\text{m.}}{\text{sec.}}$, $\frac{dY}{dt} = -2 \ \frac{\text{m.}}{\text{sec.}}$, and $\frac{dz}{dt} = 1$ m./sec.; when $x = 4$ m., $Y = 3$ m., and $z = 2$ m.:

a.) Find $\frac{dV}{dt}$: $V = XYZ \xrightarrow{D}$

$$\frac{dV}{dt} = \frac{dX}{dt} \cdot YZ + X \cdot \frac{dY}{dt} \cdot Z + XY \cdot \frac{dZ}{dt}$$

$$= (1)(3)(2) + (4)(-2)(2) + (4)(3) \cdot (1)$$

$$= 2 \text{ m.}^3/\text{sec.}$$

b.) Find $\frac{dS}{dt}$: $S = 2XY + 2YZ + 2XZ \longrightarrow$

$$\frac{dS}{dt} = 2X \cdot \frac{dY}{dt} + 2\frac{dX}{dt} \cdot Y + 2Y \cdot \frac{dZ}{dt} + 2 \cdot \frac{dY}{dt} \cdot Z + 2X \cdot \frac{dZ}{dt} + 2 \cdot \frac{dX}{dt} \cdot Z$$

$$= 2(4)(-2) + 2(1)(3) + 2(3)(1) + 2(-2)(2) + 2(4)(1) + 2(1)(2)$$

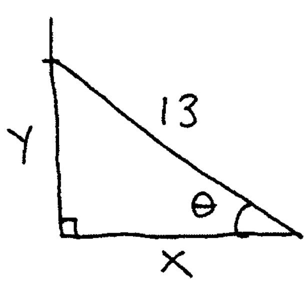
$$= -16 + 6 + 6 - 8 + 8 + 4 = 0 \text{ m.}^2/\text{sec.}$$

c.) Find $\frac{dL}{dt}$: $L = \sqrt{X^2 + Y^2 + Z^2} \xrightarrow{D}$

$$\frac{dL}{dt} = \frac{1}{\cancel{2}}(X^2+Y^2+Z^2)^{-1/2} \cdot \left[\cancel{2}X \cdot \frac{dX}{dt} + \cancel{2}Y \cdot \frac{dY}{dt} + \cancel{2}Z\frac{dZ}{dt}\right]$$

$$= (16+9+4)^{-1/2}\left[(4)(1) + (3)(-2) + (2)(1)\right] = \frac{0}{\sqrt{29}} = 0 \text{ m./sec.}$$

23.)

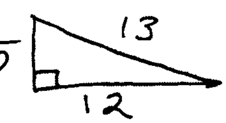


assume $\frac{dX}{dt} = 5$ ft./sec. when $X = 12$ ft. ;

a.) Find $\frac{dY}{dt}$: $X^2 + Y^2 = 13^2 \xrightarrow{D}$

$$2X \cdot \frac{dX}{dt} + 2Y\frac{dY}{dt} = 0 \longrightarrow$$

$Y = 5$



$$2(12) \cdot (5) + 2(5)\frac{dY}{dt} = 0 \longrightarrow 10 \cdot \frac{dY}{dt} = -120 \longrightarrow$$

$$\frac{dY}{dt} = -12 \text{ ft./sec.}$$

c.) Find $\frac{d\theta}{dt}$: $\tan\theta = \frac{Y}{X} \rightarrow$

$\theta = \arctan\left(Y/X\right) \xrightarrow{D}$

$$\frac{d\theta}{dt} = \frac{1}{1+\left(\frac{Y}{X}\right)^2} \cdot \frac{X \cdot \frac{dY}{dt} - Y \cdot \frac{dX}{dt}}{X^2}$$

$$= \frac{1}{1+\left(\frac{5}{12}\right)^2} \cdot \frac{(12)(-12)-(5)(5)}{(12)^2}$$

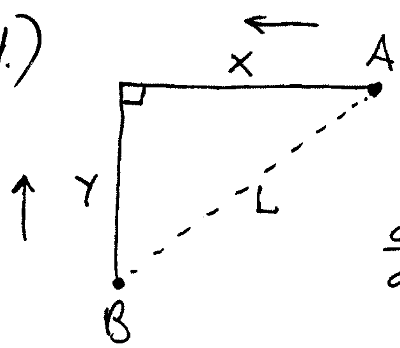
$$= \frac{144}{169} \cdot \frac{-169}{144} = -1 \text{ rad./sec.}$$

b.) Find $\frac{dA}{dt}$: $A = \frac{1}{2}XY \xrightarrow{D}$

$$\frac{dA}{dt} = \frac{1}{2}X \cdot \frac{dY}{dt} + \frac{1}{2}\frac{dX}{dt} \cdot Y = \frac{1}{2}(12)(-12) + \frac{1}{2}(5)(5)$$

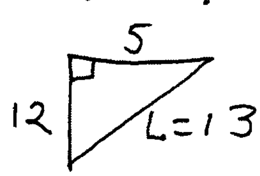
$$= -\frac{144}{2} + \frac{25}{2} = \frac{-119}{2} \text{ ft.}^2\text{/sec.}$$

24.)



assume $\frac{dX}{dt} = -442$ n. mi./hr., $\frac{dY}{dt} = -481$ n. mi./hr. ; find $\frac{dL}{dt}$ when $X = 5$ n. mi. and $Y = 12$ n. mi. :

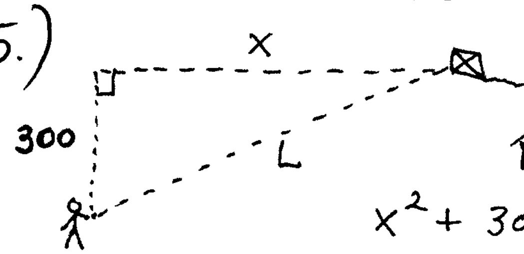
$X^2 + Y^2 = L^2 \xrightarrow{D} \cancel{2}X \cdot \frac{dX}{dt} + \cancel{2}Y \frac{dY}{dt} = \cancel{2}L \cdot \frac{dL}{dt} \rightarrow$



$(5)(-442) + (12)(-481) = (13)\,\frac{dL}{dt} \rightarrow$

$$\frac{dL}{dt} = \frac{-7982}{13} = -614 \text{ n. mi./hr.}$$

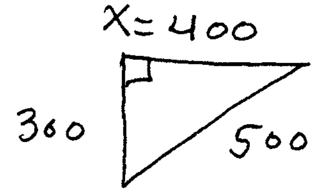
25.)



assume $\frac{dX}{dt} = 25$ ft./sec. ; find $\frac{dL}{dt}$ when $L = 500$ ft. :

$X^2 + 300^2 = L^2 \xrightarrow{D}$

$\cancel{2}x \cdot \frac{dx}{dt} = \cancel{2}L \cdot \frac{dL}{dt} \rightarrow$



$400(25) = (500) \cdot \frac{dL}{dt} \rightarrow \frac{dL}{dt} = 20$ ft./sec.

27.)

assume $\frac{dV}{dt} = 10$ m.^{3}/min

and $h = \frac{3}{8}(2r) \rightarrow \boxed{h = \frac{3}{4} r} \xrightarrow{D}$

$\boxed{\frac{dh}{dt} = \frac{3}{4} \cdot \frac{dr}{dt}}$; volume $V = \frac{1}{3}\pi r^2 h \rightarrow$

$V = \frac{1}{3}\pi r^2 \left(\frac{3}{4} r\right) \rightarrow \boxed{V = \frac{1}{4}\pi r^3}$; find $\frac{dr}{dt}$

and $\frac{dh}{dt}$ when $\underline{h = 4 \text{ m.}}$: $V = \frac{1}{4}\pi r^3 \xrightarrow{D}$

$\frac{dV}{dt} = \frac{1}{4}\pi \cdot 3r^2 \cdot \frac{dr}{dt} \rightarrow 10 = \frac{1}{4}\pi \cdot 3r^2 \cdot \frac{dr}{dt}$

$\left(h = 4 \rightarrow 4 = \frac{3}{4} r \rightarrow r = 16/3\right) \rightarrow$

$10 = \frac{3}{4}\pi \left(\frac{16}{3}\right)^2 \cdot \frac{dr}{dt} \rightarrow 10 = \frac{64}{3}\pi \frac{dr}{dt} \rightarrow$

$\boxed{\frac{dr}{dt} = \frac{15}{32\pi} \approx 0.149 \text{ m./min.}}$; then

$\frac{dh}{dt} = \frac{3}{4} \cdot \frac{dr}{dt} \approx \frac{3}{4}(0.149) \approx 0.112$ m./min. $\rightarrow$

$\boxed{\frac{dh}{dt} \approx 0.112 \text{ m./min.}}$

31.)

assume $\frac{dV}{dt} = 100\pi$ ft.^{3}/min. ;

when $r = 5$ ft. :

a.) Find $\frac{dr}{dt}$: $V = \frac{4}{3}\pi r^3 \xrightarrow{D}$

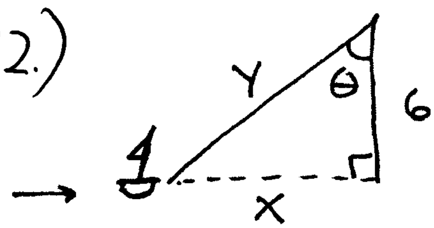
$\frac{dV}{dt} = \frac{4}{\cancel{3}}\pi \cdot \cancel{3} r^2 \cdot \frac{dr}{dt} \rightarrow$

$$100\pi = 4\pi(5)^2 \cdot \frac{dr}{dt} \to \boxed{\frac{dr}{dt} = 1 \text{ ft./min.}}$$

b.) Find $\frac{dS}{dt}$: $S = 4\pi r^2 \xrightarrow{D}$

$$\frac{dS}{dt} = 4\pi \cdot 2r \cdot \frac{dr}{dt} = 8\pi(5)\cdot(1) = 40\pi \frac{\text{ft.}^2}{\text{min.}}$$

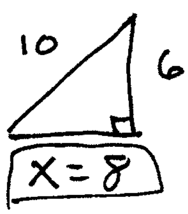
32.)



assume $\frac{dY}{dt} = -2$ ft./sec.; when $Y = 10$ ft.:

a.) Find $\frac{dx}{dt}$: $x^2 + 6^2 = Y^2 \to$

$$\cancel{2}x \cdot \frac{dx}{dt} = \cancel{2}Y \frac{dY}{dt} \to 8 \cdot \frac{dx}{dt} = 10(-2) \to$$



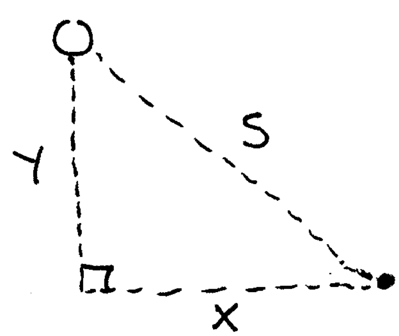
$$\frac{dx}{dt} = -\frac{20}{8} = -\frac{5}{2} \text{ ft./sec.}$$

b.) Find $\frac{d\theta}{dt}$: $\tan\theta = \frac{x}{6} \to$

$$\theta = \arctan\left(\frac{x}{6}\right) \xrightarrow{D} \frac{d\theta}{dt} = \frac{1}{1+\left(\frac{x}{6}\right)^2} \cdot \frac{1}{6}\frac{dx}{dt}$$

$$\to \frac{d\theta}{dt} = \frac{1}{1+\left(\frac{8}{6}\right)^2} \cdot \frac{1}{6} \cdot \left(-\frac{5}{2}\right) = \frac{36}{100} \cdot \frac{-5}{12} = \frac{-3}{20} \frac{\text{rad.}}{\text{sec.}}$$

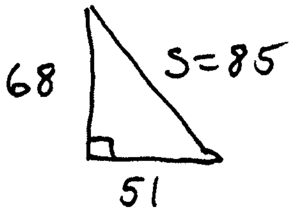
33.)



assume $\frac{dY}{dt} = 1$ ft./sec. and $\frac{dx}{dt} = 17$ ft./sec.; find $\frac{dS}{dt}$ when

$t = 3$ sec. $\to$ $x = 51$ ft. and $y = 65+3 = 68$ ft.:

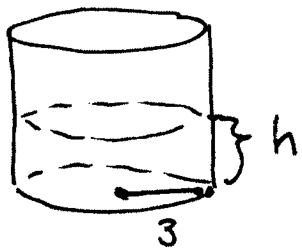
$x^2 + y^2 = s^2 \to \cancel{2}x \cdot \frac{dx}{dt} + \cancel{2}y\frac{dy}{dt} = \cancel{2}s\frac{ds}{dt}$



$\to (51)(17) + (68)(1) = (85) \cdot \frac{ds}{dt}$

$\to \frac{ds}{dt} = \frac{935}{85} = 11$ ft./sec.

34.) a.)



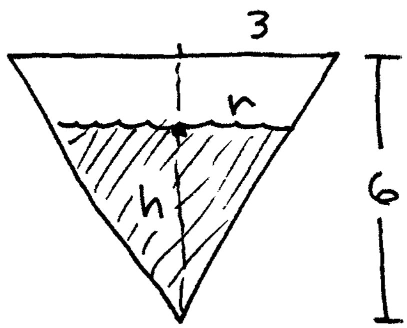
assume $\frac{dV}{dt} = 10$ in.^{3}/min.; find $\frac{dh}{dt}$:

$V = \pi r^2 h = \pi(3)^2 h \to V = 9\pi h \xrightarrow{D}$

$\frac{dV}{dt} = 9\pi\frac{dh}{dt} \to 10 = 9\pi \cdot \frac{dh}{dt} \to$

$\frac{dh}{dt} = \frac{10}{9\pi}$ in./min.

b.)



By similar triangles $\frac{r}{h} = \frac{3}{6} \to r = \frac{1}{2}h$; then volume

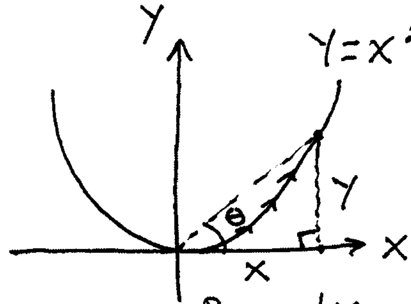
$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{1}{2}h\right)^2 h = \frac{\pi}{12}h^3 \to$

$\boxed{V = \frac{\pi}{12}h^3}$; find $\frac{dh}{dt}$ when $h = 5$ in.:

$\frac{dV}{dt} = \frac{\pi}{12} \cdot 3h^2 \cdot \frac{dh}{dt} \to -10 = \frac{\pi}{4} \cdot (5)^2 \cdot \frac{dh}{dt} \to$

$\to \frac{dh}{dt} = \frac{-40}{25\pi} = \frac{-8}{5\pi}$ in./min.

36.)



assume $\frac{dX}{dt} = 10$ m./sec. ; find $\frac{dY}{dt}$ and $\frac{d\theta}{dt}$ when $X = 3$ m. and $Y = 9$ m. :

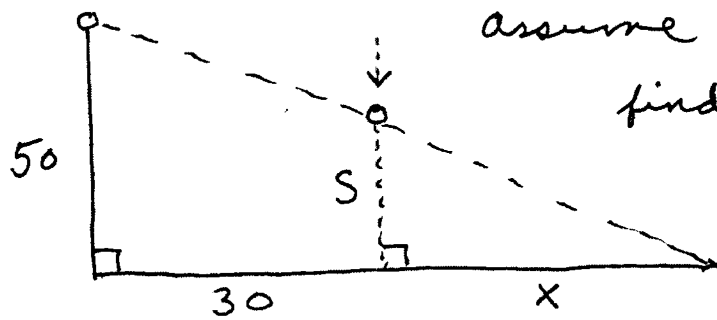
$Y = X^2 \xrightarrow{D} \frac{dY}{dt} = 2X \cdot \frac{dX}{dt} = 2(3)(10) = 60$ m./sec. ;

$\rightarrow \frac{dY}{dt} = 60$ m./sec. ; $\tan\theta = \frac{Y}{X} \rightarrow$

$$\theta = \arctan\left(\frac{Y}{X}\right) \xrightarrow{D} \frac{d\theta}{dt} = \frac{1}{1+\left(\frac{Y}{X}\right)^2} \cdot \frac{X \cdot \frac{dY}{dt} - Y \cdot \frac{dX}{dt}}{X^2}$$

$$= \frac{1}{1+\left(\frac{9}{3}\right)^2} \cdot \frac{(3)(60)-(9)(10)}{3^2} = \frac{1}{10} \cdot \frac{90}{9} = 1 \text{ rad./sec.}$$

39.)



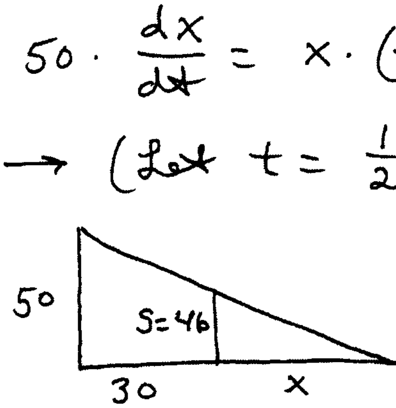
assume $S = 50 - 16t^2$; find $\frac{dX}{dt}$ when $t = \frac{1}{2}$ sec. : by similar triangles

$$\frac{50}{X+30} = \frac{S}{X} \rightarrow 50X = X \cdot S + 30S \xrightarrow{D}$$

$$50 \cdot \frac{dX}{dt} = X \cdot \frac{dS}{dt} + \frac{dX}{dt} \cdot S + 30 \cdot \frac{dS}{dt} \longrightarrow$$

$$50 \cdot \frac{dX}{dt} = X \cdot (-32t) + \frac{dX}{dt} \cdot (50 - 16t^2) + 30(-32t)$$

$\rightarrow$ (Let $t = \frac{1}{2} \rightarrow S = 46$ ft; then



$\frac{50}{30+X} = \frac{46}{X} \rightarrow 50X = 1380 + 46X \rightarrow$
$4X = 1380 \rightarrow X = 345$ ft.) $\longrightarrow$

$$50 \cdot \frac{dX}{dt} = (345)(-16) + \frac{dX}{dt}(46) - 480 \longrightarrow$$

$$4 \cdot \frac{dX}{dt} = -6000 \longrightarrow \frac{dX}{dt} = -1500 \text{ ft./sec.}$$