Section 3.11

1.) $f(x) = x^3 - 2x + 3$, $a = 2$

x-values : $2 \to x$, $f'(x) = 3x^2 - 2$;

$\Delta f = f(x) - f(2) = f(x) - 7$; $\Delta x = x - 2$ so

$df = f'(2) \cdot \Delta x = (10) \cdot (x-2) = 10x - 20$;

assume $\Delta f \approx df$ $\longrightarrow$

$f(x) - 7 \approx 10x - 20 \longrightarrow f(x) \approx \underbrace{10x - 13}_{L(x)}$

3.) $f(x) = x + \frac{1}{x}$, $a = 1$, x-values : $1 \to x$

so $\Delta x = x - 1$, $f'(x) = 1 - \frac{1}{x^2}$;

$\Delta f = f(x) - f(1) = f(x) - 2$,

$df = f'(1) \cdot \Delta x = (0) \cdot (x-1) = 0$; assume

$\Delta f \approx df \longrightarrow f(x) - 2 \approx 0 \longrightarrow f(x) \approx \underbrace{2}_{L(x)}$

6.) a.) $f(x) = \sin x$, $a = 0$,

x-values : $0 \to x$, $f'(x) = \cos x$;

$\Delta f = f(x) - f(0) = f(x) - 0 = f(x)$; $\Delta x = x - 0 = x$;

$df = f'(0) \cdot \Delta x = (1) \cdot x = x$; assume

$\Delta f \approx df \longrightarrow f(x) \approx \underbrace{x}_{L(x)}$

d.) $f(x) = e^x$, $a = 0$, x-values : $0 \to x$

so $\Delta x = x - 0 = x$, $f'(x) = e^x$;

$\Delta f = f(x) - f(0) = f(x) - 1$,

$df = f'(0) \cdot \Delta x = (1) \cdot x = x$; assume

$\Delta f \approx df \longrightarrow f(x) - 1 \approx x \longrightarrow f(x) \approx \underbrace{x + 1}_{L(x)}$

e.) $f(x) = \ln(1+x)$, $a=0$, x-values: $0 \to x$
so $\Delta x = x - 0 = x$, $f'(x) = \frac{1}{1+x}$;
$\Delta f = f(x) - f(0) = f(x) - 0 = f(x)$,
$df = f'(0) \cdot \Delta x = (1) \cdot x = x$; assume
$\Delta f \approx df \to f(x) \approx \underbrace{x}_{L(x)}$

15.) $f(x) = (1+x)^k$, $a=0$, x-values: $0 \to x$
so $\Delta x = x - 0 = x$, $f'(x) = k(1+x)^{k-1}$;
$\Delta f = f(x) - f(0) = f(x) - 1$,
$df = f'(0) \cdot \Delta x = k \cdot x$; assume $\Delta f \approx df \to$
$f(x) - 1 \approx kx \to f(x) \approx \underbrace{1 + kx}_{L(x)}$

16.) $\underline{L(x) = 1 + kx \approx (1+x)^k}$:
a.) $f(x) = (1-x)^6 = (1 + (-x))^6$
$\approx 1 + 6(-x) = 1 - 6x$

c.) $f(x) = \frac{1}{\sqrt{1+x}} = (1+x)^{-1/2}$
$\approx 1 + (\frac{-1}{2})x = 1 - \frac{1}{2}x$

d.) $f(x) = \sqrt{2 + x^2} = \sqrt{2(1 + \frac{x^2}{2})} = \sqrt{2}\sqrt{1 + \frac{x^2}{2}}$
$= \sqrt{2} \cdot \left(1 + \left(\frac{x^2}{2}\right)\right)^{1/2}$
$\approx \sqrt{2}\left(1 + \frac{1}{2}\left(\frac{x^2}{2}\right)\right) = \sqrt{2}\left(1 + \frac{1}{4}x^2\right)$

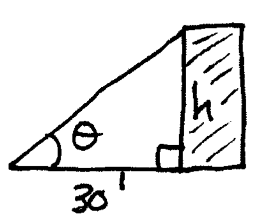
17.) a.) $(1+x)^k \approx 1 + kx$ so $(1.0002)^{50} = (1 + 0.0002)^{50}$

$\approx 1 + 50(0.0002) = 1 + 0.01 = 1.01$

51.) $A = \pi r^2$, $r: 2 \to 2.02$ m. so
$\Delta r = 0.02$, $A' = 2\pi r$,
a.) $\Delta A \approx dA = A'(2) \cdot \Delta r = 4\pi(0.02)$
$= 0.08\pi$ m.^{2}
b.) $\%$ err. $= \frac{\Delta A}{A} \approx \frac{dA}{A} = \frac{0.08\cancel{\pi}}{\cancel{\pi}(2)^2} = \frac{0.08}{4}$
$= 0.02 = 2\%$

52.) a.) $C = \pi d \to$ diameter $d = \frac{1}{\pi} C$,
$d = 10$ in. $\to C = 10\pi$ in.; $C: 10\pi \to 10\pi + 2$
so $\Delta C = 2$ in; $d' = \frac{1}{\pi}$ then
$\Delta d \approx d(d) = d'(10\pi) \cdot \Delta C = \frac{1}{\pi} \cdot 2 = \frac{2}{\pi}$ in.
b.) area $A = \pi r^2 = \pi\left(\frac{1}{2} d\right)^2 = \pi\left(\frac{1}{2}\left(\frac{1}{\pi} C\right)\right)^2 \to$
$A = \frac{1}{4\pi} C^2$ and $C: 10\pi \to 10\pi + 2$ so
$\Delta C = 2$ in.; $A' = \frac{1}{2\pi} C$ then
$\Delta A \approx dA = A'(10\pi) \cdot \Delta C = (5) \cdot 2 = 10$ in.^{2}

53.) $V = \pi r^2 h = \pi r^2 (30) = 30\pi r^2$,
$r: 6 \to 6.5$ in so $\Delta r = \frac{1}{2}$ in.,
$V' = 60\pi r$; then
$\Delta V \approx dV = V'(6) \cdot \Delta r = 360\pi \cdot \left(\frac{1}{2}\right) = 180\pi$ in.^{3}

54.) 

$\tan\theta = \frac{h}{30} \to h = 30\tan\theta$

θ-values : $\theta \to \theta + \Delta\theta$ (θ in radians!)
what must $|\Delta\theta|$ be in order that
$\frac{|\Delta h|}{h} \le 4\%$? Then

$$\frac{|\Delta h|}{h} \approx \frac{|dh|}{h} = \frac{|h'(\theta)\cdot\Delta\theta|}{h} = \frac{|\cancel{30}\sec^2\theta\cdot\Delta\theta|}{\cancel{30}\tan\theta}$$

$$= \frac{\cancel{\cos\theta}}{\sin\theta}\cdot\frac{1}{\cos^{\cancel{2}}\theta}\cdot|\Delta\theta| \qquad (\theta = 75^\circ)$$

$$= \frac{1}{\sin 75^\circ\cdot\cos 75^\circ}\cdot|\Delta\theta| \le 4\% \to$$

$$|\Delta\theta| \le 4\%(\sin 75^\circ \cos 75^\circ) = 0.04\left(\tfrac{1}{4}\right) = 0.01$$

$\to \boxed{|\Delta\theta| \le 0.01 \text{ radians}}$ ($75^\circ \approx 1.309$ rad)

% error: $\frac{|\Delta\theta|}{\theta} \le \frac{0.01}{1.309} \approx$ 0.76%

55.) $V = \pi h^3$, $V' = 3\pi h^2$; h-values : $h \to h + \Delta h$, assume $\frac{|\Delta V|}{V} \le 1\%$,
find $\frac{|\Delta h|}{h}$:

$$\frac{|\Delta V|}{V} \approx \frac{|dV|}{V} = \frac{|V'(h)\cdot\Delta h|}{V} = \frac{3\cancel{\pi}h^2\cdot|\Delta h|}{\cancel{\pi}h^3}$$

$$= 3\cdot\frac{|\Delta h|}{h} \le 1\% \to \frac{|\Delta h|}{h} \le \frac{1}{3} \text{ of } 1\% .$$

1.) Estimate $\sqrt{90}$: Let $f(x) = \sqrt{x}$,

$x: 100 \longrightarrow 90$ so $\Delta x = -10$, $f'(x) = \frac{1}{2\sqrt{x}}$;

$\Delta f = f(90) - f(100) = \sqrt{90} - 10$,

$df = f'(100) \cdot \Delta x = \frac{1}{20}(-10) = -\frac{1}{2}$; assume

$\Delta f \approx df \longrightarrow \sqrt{90} - 10 \approx -\frac{1}{2} \longrightarrow$

$\boxed{\sqrt{90} \approx 9.5}$; calc : $\sqrt{90} \approx 9.487$ so

$\%\ err. = \frac{9.5 - 9.487}{9.487} \approx 0.0014 = 0.14\%$.

2.) Estimate 2.0002^{25} : Let $f(x) = x^{25}$,

$x: 2 \rightarrow 2.0002$ so $\Delta x = 0.0002$,

$f'(x) = 25x^{24}$; then

$\Delta f = f(2.0002) - f(2) = 2.0002^{25} - 33{,}554{,}432$;

$df = f'(2) \cdot \Delta x = (419{,}430{,}400) \cdot (0.0002) = 83{,}886$;

assume $\Delta f \approx df \longrightarrow$

$2.0002^{25} - 33{,}554{,}432 \approx 83{,}886 \longrightarrow$

$\boxed{2.0002^{25} \approx 33{,}638{,}318}$;

calc: $2.0002^{25} \approx 33{,}638{,}418$ so

$\%\ err. = \frac{33{,}638{,}418 - 33{,}638{,}318}{33{,}638{,}418}$

$\approx 0.000003 = 0.0003\%$

3.) Estimate $\cos(0.01)$: Let $f(x) = \cos x$,
$x: 0 \to 0.01$ so $\Delta x = 0.01$, $f'(x) = -\sin x$;
$\Delta f = f(0.01) - f(0) = \cos(0.01) - 1$,
$df = f'(0) \cdot \Delta x = (0) \cdot (0.01) = 0$; assume
$\Delta f \approx df \to \cos(0.01) - 1 \approx 0 \to$
$\boxed{\cos(0.01) \approx 1}$;

calc : $\cos(0.01) \approx 0.99995$ so

$$\% \text{ err.} = \frac{1 - 0.99995}{0.99995} \approx 0.00005 = 0.005\%$$