Section 4.1

53.) $Y = 2X^2 - 8X + 9 \xrightarrow{D} Y' = 4X - 8 = 4(X-2) = 0$
$\rightarrow X = 2$

$-$ 0 $+$ $\quad Y'$

abs. min. $\begin{cases} X = 2 \\ Y = 1 \end{cases}$

57.) $Y = \sqrt{X^2-1} \xrightarrow{D} Y' = \frac{1}{2}(X^2-1)^{-1/2} \cdot 2X = \frac{X}{\sqrt{X^2-1}}$
$= 0 \rightarrow X = 0$
(NOT in domain!)

$-$ NO NO NO $+$ $\quad Y'$

abs. min. $\begin{cases} X = -1 \\ Y = 0 \end{cases}$ $\quad \begin{matrix} X = 1 \\ Y = 0 \end{matrix}\Big\}$ abs. min.

61.) $Y = \frac{X}{X^2+1} \xrightarrow{D} Y' = \frac{(X^2+1)(1) - X(2X)}{(X^2+1)^2}$
$= \frac{1-X^2}{(X^2+1)^2} = 0 \rightarrow 1 - X^2 = 0 \rightarrow X = 1, X = -1$

$-$ 0 $+$ 0 $-$ $\quad Y'$

Y is $-$ $\quad X = -1 \quad X = 1 \quad$ Y is $+$

$\underbrace{Y = -1/2}$ $\quad \underbrace{Y = 1/2}$

abs. min. $\quad$ abs. max.

63.) $Y = e^X + e^{-X} \xrightarrow{D} Y' = e^X - e^{-X} = e^X - \frac{1}{e^X}$
$= \frac{e^{2X}-1}{e^X} = 0 \rightarrow e^{2X} - 1 = 0 \rightarrow e^{2X} = 1 \rightarrow$
$2X = 0 \rightarrow X = 0$

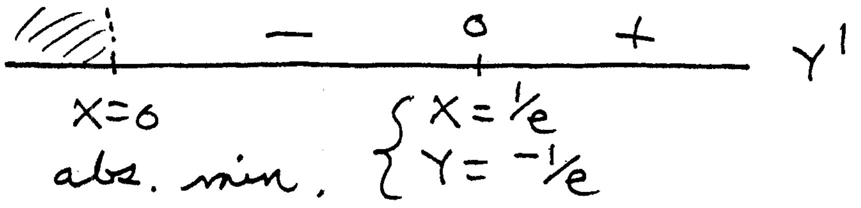
$-$ 0 $+$ $\quad Y'$

abs. min. $\begin{cases} X = 0 \\ Y = 2 \end{cases}$

65.) $Y = x \ln x, \quad x > 0 \quad \xrightarrow{D}$

$Y' = x \cdot \frac{1}{x} + (1) \ln x = 1 + \ln x = 0 \rightarrow \ln x = -1$

$\rightarrow x = e^{-1} = 1/e$



Section 4.3

4.) $f'(x) = (x-1)^2(x+2)^2 = 0 \longrightarrow x=1, x=-2$

$$\begin{array}{ccccc} + & 0 & + & 0 & + \\ \hline & x=-2 & & x=1 & \end{array} \quad f'$$

f is ↑ for $x<-2$, $-2<x<1$, $x>1$

5.) $f'(x) = (x-1)e^{-x} = 0 \longrightarrow x = 1$

$$\begin{array}{ccc} - & 0 & + \\ \hline & \underbrace{x=1}_{\text{abs. min.}} & \end{array} \quad f'$$

f is ↑ for $x>1$, f is ↓ for $x<1$

28) $g(x) = x^4 - 4x^3 + 4x^2 \xrightarrow{D}$

$g'(x) = 4x^3 - 12x^2 + 8x = 4x(x^2 - 3x + 2)$

$= 4x(x-1)(x-2) = 0 \longrightarrow x=1, x=2, x=0$

$$\begin{array}{ccccccc} - & 0 & + & 0 & - & 0 & + \\ \hline & x=0 & & x=1 & & x=2 & \\ & \underbrace{y=0}_{\text{abs. min}} & & \underbrace{y=1}_{\substack{\text{rel.}\\ \text{max}}} & & \underbrace{y=0}_{\text{abs. min.}} & \end{array} \quad g'$$

g is ↑ for $0<x<1$, $x>2$;
g is ↓ for $x<0$, $1<x<2$

33.) $g(x) = x\sqrt{8-x^2} \longrightarrow -\sqrt{8} \le x \le \sqrt{8}$ and

$g'(x) = x \cdot \frac{1}{\cancel{2}}(8-x^2)^{-1/2} \cdot (-\cancel{2}x) + (1)\cdot\sqrt{8-x^2}$

$$= \frac{-x^2}{\sqrt{8-x^2}} + \frac{\sqrt{8-x^2}}{1} = \frac{-x^2 + (8-x^2)}{\sqrt{8-x^2}}$$

$$= \frac{8-2x^2}{\sqrt{8-x^2}} = \frac{2(2-x)(2+x)}{\sqrt{8-x^2}} = 0 \rightarrow$$

$2(2-x)(2+x) = 0 \rightarrow x = 2, x = -2$

g': ▨ | $-$ 0 $+$ 0 $-$ | ▨

$x=-\sqrt{8}$	$x=-2$	$x=2$	$x=\sqrt{8}$
$\underbrace{Y=0}$	$\underbrace{Y=-4}$	$\underbrace{Y=4}$	$\underbrace{Y=0}$
rel.	abs.	abs.	rel.
max.	min.	max	min.

g is ↑ for $-2 < x < 2$;

g is ↓ for $-\sqrt{8} < x < -2,\ 2 < x < \sqrt{8}$

41.) $f(x) = e^{2x} + e^{-x} \xrightarrow{D}$

$f'(x) = 2e^{2x} - e^{-x} = \frac{2e^{2x}}{1} - \frac{1}{e^x}$

$= \frac{2e^{3x}-1}{e^x} = 0 \rightarrow 2e^{3x} - 1 = 0 \rightarrow$

$e^{3x} = 1/2 \rightarrow \ln e^{3x} = \ln(1/2) = \overset{0}{\cancel{\ln 1}} - \ln 2 \rightarrow$

$3x = -\ln 2 \rightarrow x = \frac{-1}{3}\ln 2$

f': $-$ 0 $+$

$x = \frac{-1}{3}\ln 2 \approx -0.231$

$Y = e^{-\frac{2}{3}\ln 2} + e^{\frac{1}{3}\ln 2} = e^{\ln 2^{-2/3}} + e^{\ln 2^{1/3}}$ } abs. min.

$= \frac{1}{2^{2/3}} + \frac{2^{1/3}}{1} = \frac{1+2}{2^{2/3}} = \frac{3}{2^{2/3}}$

f is ↑ for $x > -1/3 \ln 2$;

f is ↓ for $x < \frac{-1}{3}\ln 2$