Section 4.4

1.) $y = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + \frac{1}{3}$, Domain: all x-values

$y' = x^2 - x - 2 = (x-2)(x+1) = 0 \rightarrow x = -1, x = 2$

	+	0	−	0	+	y'

rel. max. $x = -1$, $y = 3/2$; rel. min. $x = 2$, $y = -3$

$y'' = 2x - 1 = 0 \rightarrow x = \frac{1}{2}$

	−	0	+	y''

infl. pt. $x = \frac{1}{2}$, $y = -3/4$

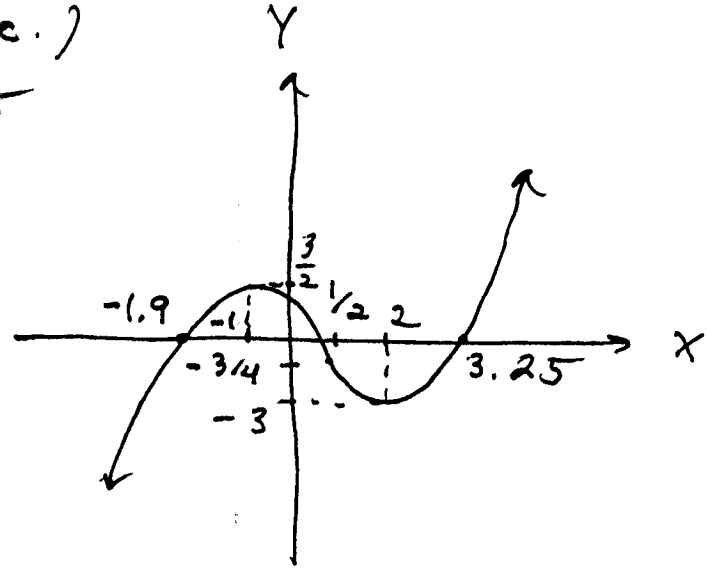
y is ↑ for $x < -1$, $x > 2$;
y is ↓ for $-1 < x < 2$;
y is ∪ for $x > \frac{1}{2}$;
y is ∩ for $x < \frac{1}{2}$;

$x = 0$: $y = \frac{1}{3}$

$y = 0$: $\frac{x^3}{3} - \frac{x^2}{2} - 2x + \frac{1}{3} = 0$

(use graphing calc.)

$x \approx -1.9, 0.15, 3.25$



2.) $Y = \frac{x^4}{4} - 2x^2 + 4$, Domain: all x-values,

$Y' = x^3 - 4x = x(x-2)(x+2) = 0 \rightarrow$

$x = 0,\ x = 2,\ x = -2$

$-$ 0 $+$ 0 $-$ 0 $+$ Y'

$x = -2$ $x = 0$ $x = 2$

$Y = 0$ $Y = 4$ $Y = 0$

abs. min. rel. max. abs. min.

$Y'' = 3x^2 - 4 = 0 \rightarrow x = \pm 2/\sqrt{3}$

$+$ 0 $-$ 0 $+$ Y''

infl. pt. $\left\{ x = -\frac{2}{\sqrt{3}},\ Y = \frac{16}{9} \right.$ $\left. x = 2/\sqrt{3},\ Y = \frac{16}{9} \right\}$ infl. pt.

Y is ↑ for $-2 < X < 0$, $X > 2$;

Y is ↓ for $X < -2$, $0 < X < 2$;

Y is ∪ for $X < -2/\sqrt{3}$, $X > 2/\sqrt{3}$;

Y is ∩ for $-\frac{2}{\sqrt{3}} < X < \frac{2}{\sqrt{3}}$

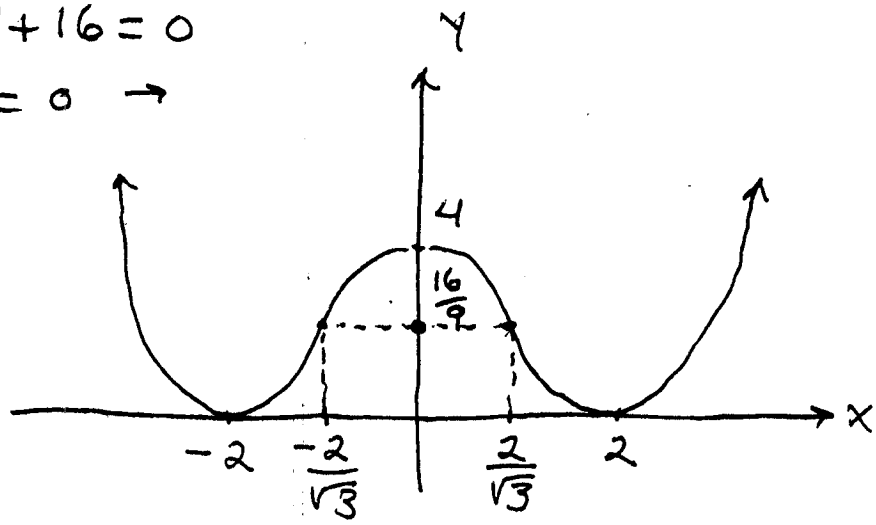
$X = 0$: $Y = 4$

$Y = 0$: $\frac{1}{4}(x^2)^2 - 2(x^2) + 4 = 0$

$\rightarrow (x^2)^2 - 8(x^2) + 16 = 0$

$(x^2 - 4)(x^2 + 4) = 0 \rightarrow$

$x = 2,\ x = -2$



10.) $Y = 6 - 2X - X^2$, Domain: all x-values,

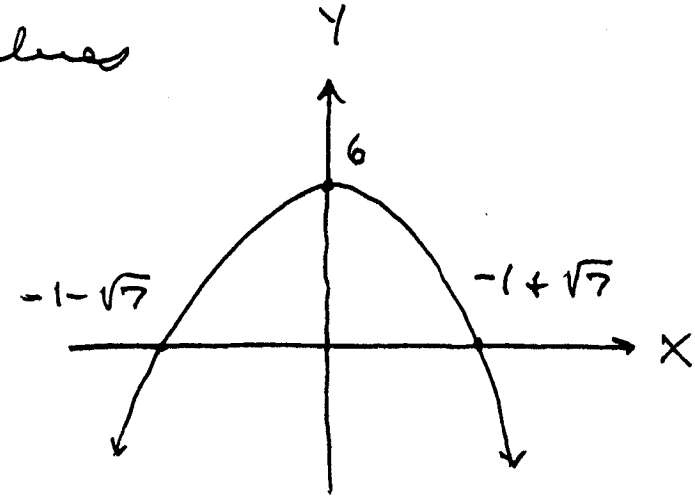
$Y' = -2 - 2X = -2(1+X) = 0 \rightarrow X = -1$

$+ \quad 0 \quad - \qquad Y'$

abs. max. $\{ X = -1, \; Y = 7$

$Y'' = -2 \qquad - \quad - \quad - \qquad Y''$

Y is ↑ for $X < -1$;
Y is ↓ for $X > -1$;
Y is ∩ for all x-values



$X = 0: \; Y = 6$

$Y = 0: \; X^2 + 2X - 6 = 0$

$\rightarrow$

$$X = \frac{-2 \pm \sqrt{4 - (-24)}}{2}$$

$$= \frac{-2 \pm 2\sqrt{7}}{2}$$

$$= -1 \pm \sqrt{7}$$

17.) $Y = X^4 - 2X^2$, Domain: all x-values,

$Y' = 4X^3 - 4X = 4X(X-1)(X+1) = 0 \rightarrow$

$X = 0, \; X = 1, \; X = -1$

$- \quad 0 \quad + \quad 0 \quad - \quad 0 \quad + \qquad Y'$

$X = -1$	$X = 0$	$X = 1$
$Y = -1$	$Y = 0$	$Y = -1$
abs. min.	rel. max.	abs. min.

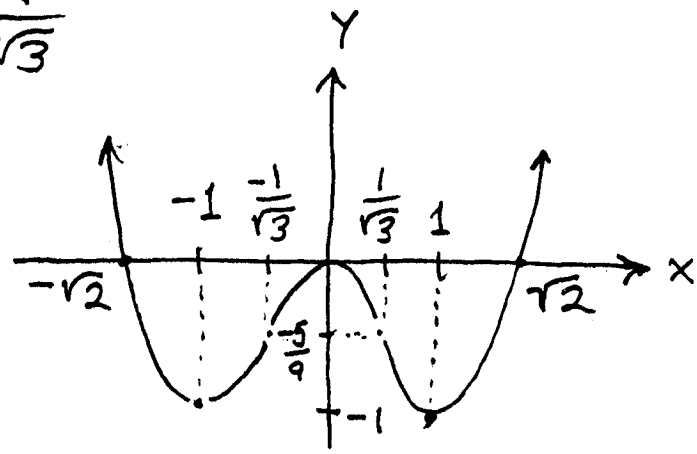
$Y'' = 12x^2 - 4 = 4(3x^2 - 1) = 0 \rightarrow x = \pm \frac{1}{\sqrt{3}}$

$+ \quad 0 \quad - \quad 0 \quad +$ Y''

infl. pt. $\left\{ x = \frac{-1}{\sqrt{3}},\ Y = -\frac{5}{9} \right\}$ $\left\{ x = \frac{1}{\sqrt{3}},\ Y = \frac{5}{9} \right\}$ infl. pt.

Y is ↑ for $-1 < x < 0$, $x > 1$;
Y is ↓ for $x < -1$, $0 < x < 1$;
Y is ∪ for $x < -1/\sqrt{3}$, $x > 1/\sqrt{3}$;
Y is ∩ for $\frac{-1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$

$x = 0 : Y = 0$
$Y = 0 : x^4 - 2x^2 = 0$
$\rightarrow x^2(x^2 - 2) = 0$
$\rightarrow x = 0,\ x = \pm\sqrt{2}$



21.) $Y = x^5 - 5x^4$ Domain: all x-values,

$Y' = 5x^4 - 20x^3 = 5x^3(x - 4) = 0$

$+ \quad 0 \quad - \quad 0 \quad +$ Y'

rel. max. $\left\{ x = 0,\ Y = 0 \right\}$ $\left\{ x = 4,\ Y = -256 \right\}$ rel. min.

$Y'' = 20x^3 - 60x^2 = 20x^2(x - 3) = 0$

$- \quad 0 \quad - \quad 0 \quad +$ Y''

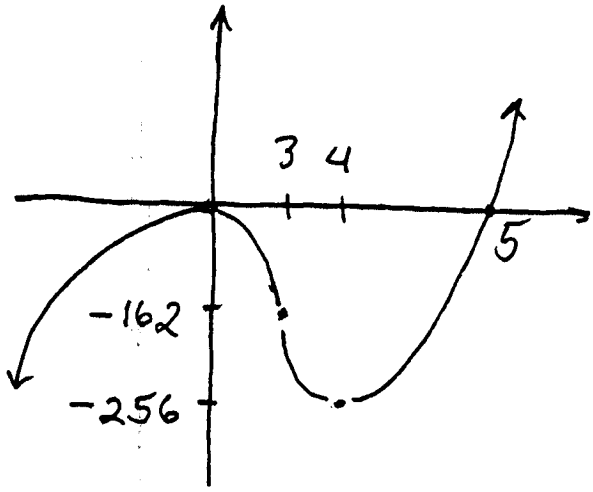
$x = 0$ $\left\{ x = 3,\ Y = -162 \right\}$ infl. pt.

$x=0: y=0$

$y=0: x^5-5x^4=0$

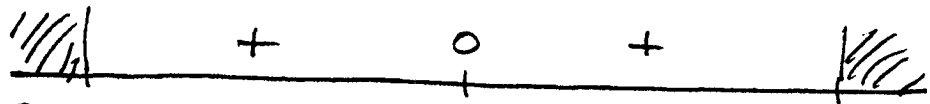
$\rightarrow x^4(x-5)=0$

$\rightarrow x=0, x=5$



23) $y=x+\sin x \qquad 0\le x\le 2\pi$

$y'=1+\cos x=0 \overset{?}{\rightarrow} \cos x=-1 \rightarrow x=\pi$



abs. min. $\begin{cases} x=0 \\ y=0 \end{cases}$ $x=\pi$ $\begin{cases} x=2\pi \\ y=2\pi \end{cases}$ abs. max.

$y''=-\sin x=0 \rightarrow x=0, \pi, 2\pi$

$x=0$ infl. pt. $\begin{cases} x=\pi \\ y=\pi \end{cases}$ $x=2\pi$

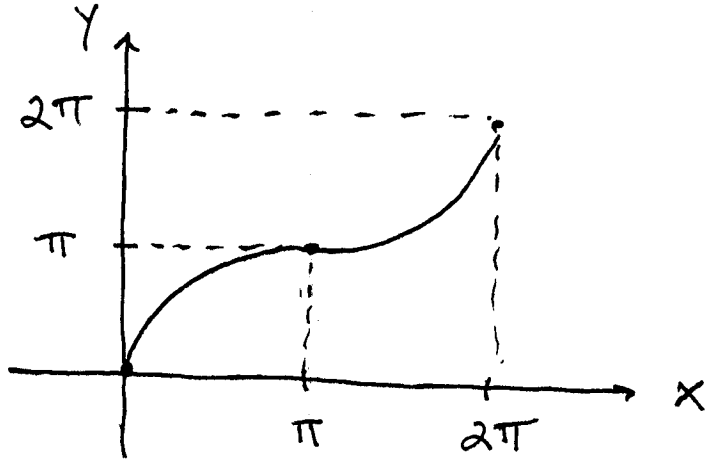
y is ↑ for $0<x<\pi$, $\pi<x<2\pi$;

y is ∪ for $\pi<x<2\pi$;

y is ∩ for $0<x<\pi$

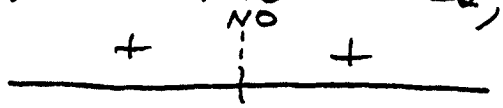
$x=0: y=0$

$y=0: x=0$



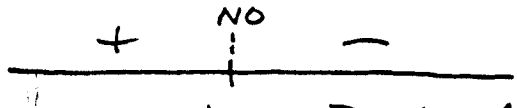
29) $y = x^{1/5}$ Domain: all x-values,

$y' = \frac{1}{5}x^{-4/5} = \frac{1}{5x^{4/5}}$



$x=0$ } vert.
$y=0$ } tan.

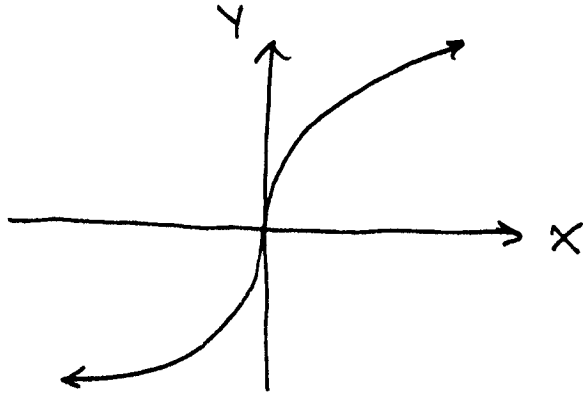
$y'' = \frac{-4}{25}x^{-9/5} = \frac{-4}{25x^{9/5}}$



$x=0$ } infl.
$y=0$ } pt.

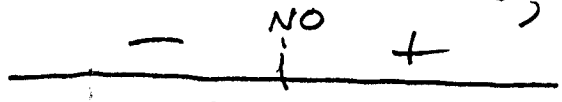
y is ↑ for $x<0$, $x>0$;
y is ∪ for $x<0$;
y is ∩ for $x>0$;

$x=0 : y=0$
$y=0 : x=0$



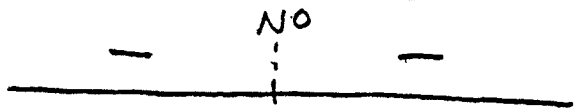
30) $y = x^{3/5}$ Domain: all x-values,

$y' = \frac{2}{5}x^{-3/5} = \frac{2}{5x^{3/5}}$



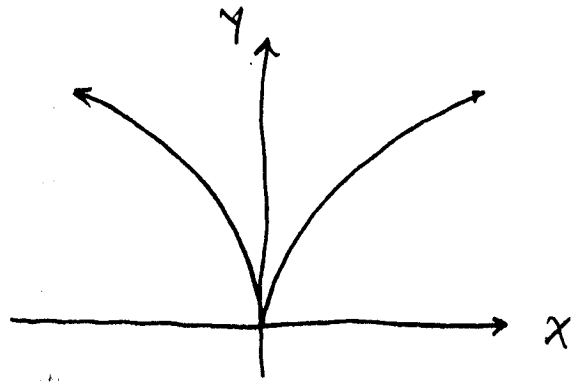
corner { $x=0$, $y=0$ } abs. min.

$y'' = \frac{-6}{25}x^{-8/5} = \frac{-6}{25x^{8/5}}$



y is ↑ for $x>0$,
y is ↓ for $x<0$,
y is ∩ for $x<0$, $x>0$,

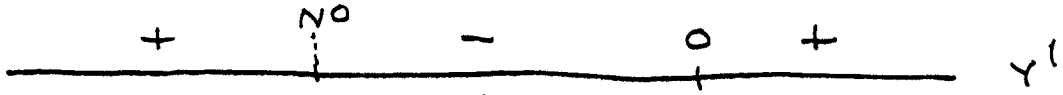
$x=0 : y=0$
$y=0 : x=0$



33) $Y = 2X - 3X^{2/3}$ Domain: all x-values,

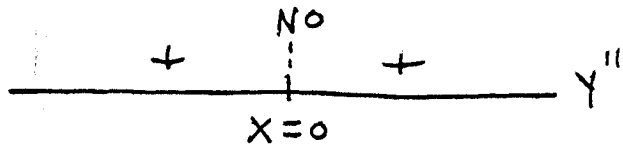
$Y' = 2 - \cancel{3}\cdot\frac{2}{\cancel{3}}X^{-1/3} = 2 - \frac{2}{X^{1/3}} = 2\left(\frac{X^{1/3}-1}{X^{1/3}}\right) = 0$

$\rightarrow X^{1/3} - 1 = 0 \rightarrow X = 1$



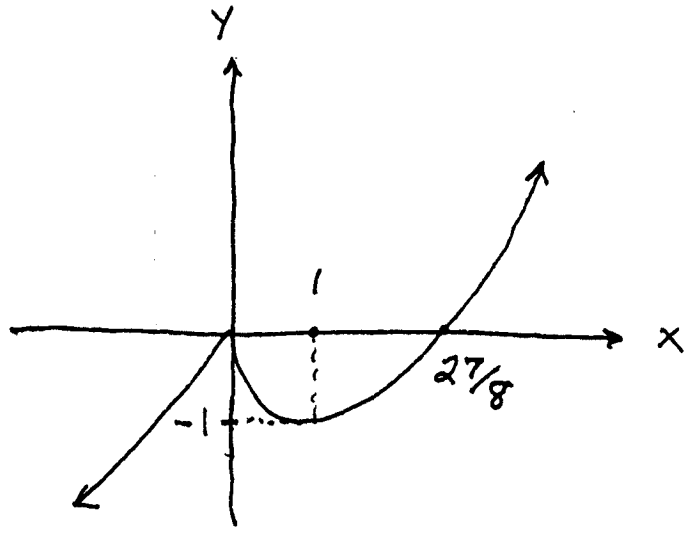
corner $\{X=0\}$ rel. max. $\{Y=0\}$ $\quad$ $X=1$ $\{Y=-1\}$ rel. min.

$Y'' = -2\cdot\frac{-1}{3}X^{-4/3} = \frac{2}{3X^{4/3}} = 0$



Y is ↑ for $x<0$, $x>1$,
Y is ↓ for $0<x<1$,
Y is ∪ for $x<0$, $x>0$;

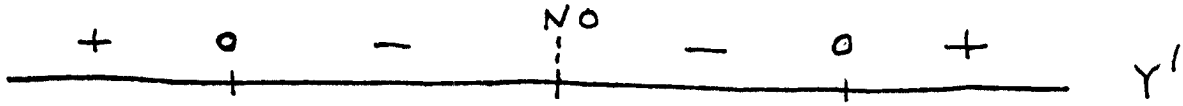
$X=0$: $Y=0$
$Y=0$: $2X - 3X^{2/3} = 0$
$\rightarrow X^{2/3}(2X^{1/3} - 3) = 0$
$\rightarrow X = 0,\ X^{1/3} = 3/2$
$\rightarrow X = {}^{27}/_{8}$



41) $Y = \frac{X^2-3}{X-2}$, Domain: all $x \neq 2$,

$Y' = \frac{(X-2)\cdot 2X - (X^2-3)(1)}{(X-2)^2} = \frac{2X^2 - 4X - X^2 + 3}{(X-2)^2}$

$= \frac{X^2-4X+3}{(X-2)^2} = \frac{(X-3)(X-1)}{(X-2)^2} = 0$



rel. max. $\{X=1, Y=2\}$ $\quad$ $X=2$ $\quad$ $\{X=3, Y=6\}$ rel. min.

$$Y'' = \frac{(x-2)^2 \cdot (2x-4) - (x^2-4x+3)\cdot 2(x-2)}{(x-2)^2}$$

$$= \frac{2(x-2)\left[\cancel{x^2} - \cancel{4x} + 4 - \cancel{x^2} + \cancel{4x} - 3\right]}{(x-2)^2} = \frac{2}{x-2} = 0$$

− NO + (sign chart for Y', break at $x = 2$)

$x = 0: \; Y = 3/2$

$Y = 0: \; x^2 - 3 = 0 \longrightarrow x = \pm\sqrt{3}$

$$\lim_{x\to\pm\infty} \frac{x^2-3}{x-2} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to\pm\infty} \frac{x - 3/x}{1 - 2/x} = \pm\infty,$$

$$\lim_{x\to 2^+} \frac{x^2-3}{x-2} = \text{"}\frac{1}{0^+}\text{"} = +\infty,$$

$$\lim_{x\to 2^-} \frac{x^2-3}{x-2} = \text{"}\frac{1}{0^-}\text{"} = -\infty, \quad \underline{V.A.: \; x = 2};$$

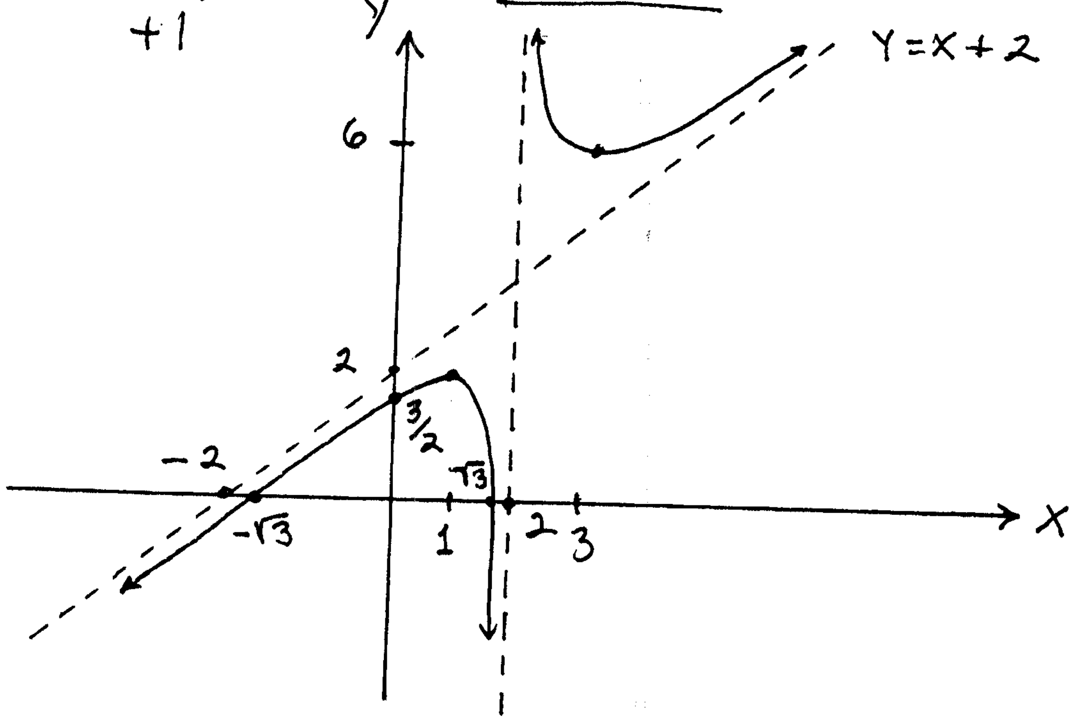
Long division of $x^2 - 3$ by $x - 2$:

$$\begin{array}{r} x + 2 \\ x-2 \overline{\big)\, x^2 - 3} \\ -(x^2 - 2x) \\ \hline 2x - 3 \\ -(2x - 4) \\ \hline +1 \end{array}$$

so $\dfrac{x^2-3}{x-2} = x + 2 + \dfrac{1}{x-2}$

so tilted asymptote is

$\underline{Y = x + 2}$

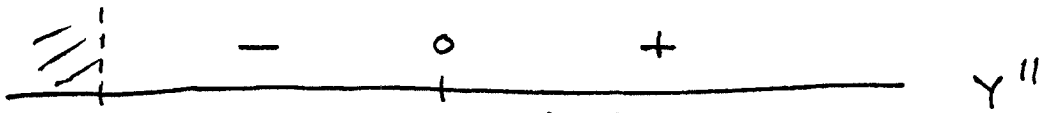


52.) $y = x(\ln x)^2$, Domain : $x > 0$,

$y' = x \cdot 2\ln x \cdot \frac{1}{x} + (\ln x)^2 = \ln x(2 + \ln x) = 0$

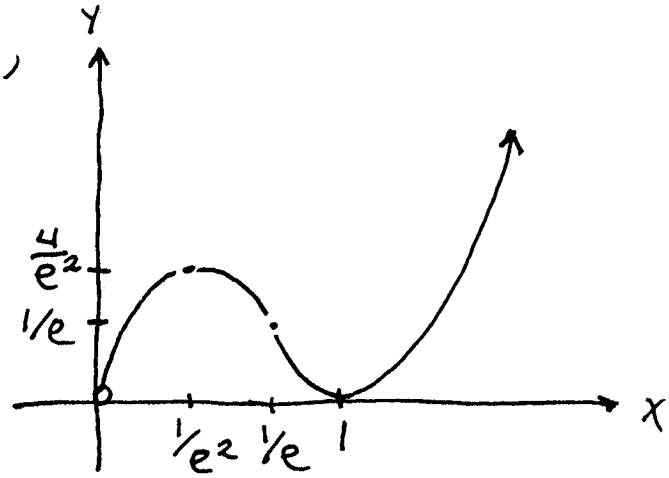
rel. max. $\{x = e^{-2}, y = 4e^{-2}$; $x = 1, y = 0\}$ abs. min.

$y'' = 2 \cdot \frac{1}{x} + 2\ln x \cdot \frac{1}{x} = 2 \cdot \frac{1}{x}(1 + \ln x) = 0$



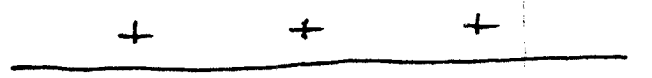
$\{x = e^{-1}, y = e^{-1}\}$ infl. pt.

y is ↑ for $0 < x < \frac{1}{e^2}$, $x > 1$,
y is ↓ for $\frac{1}{e^2} < x < 1$,
y is ∪ for $x > \frac{1}{e}$,
y is ∩ for $0 < x < \frac{1}{e}$;
$x = 0$: (No)
$y = 0$: $x = 1$



58.) $y = \frac{e^x}{1 + e^x}$, Domain : all x-values,

$y' = \frac{(1 + e^x) \cdot e^x - e^x \cdot e^x}{(1 + e^x)^2} = \frac{e^x}{(1 + e^x)^2} = 0$



$y'' = \frac{(1 + e^x)^2 \cdot e^x - e^x \cdot 2(1 + e^x) \cdot e^x}{(1 + e^x)^4}$

$$= \frac{e^x(1+e^x)\cdot[1+e^x-2e^x]}{(1+e^x)^{4\,3}} = \frac{e^x(1-e^x)}{(1+e^x)^3} = 0$$

$\rightarrow 1-e^x = 0 \rightarrow e^x = 1 \rightarrow x = 0$

$+ \quad 0 \quad -$ Y''

$\left.\begin{array}{l} x=0 \\ Y=\frac{1}{2} \end{array}\right\}$ infl. pt.

Y is ↑ for all x-values,

Y is ∪ for $x<0$,

Y is ∩ for $x>0$;

$x=0: \quad Y=\frac{1}{2}$

$Y=0: \quad$ (NO)

$$\lim_{x\to\infty} \frac{e^x}{1+e^x}\cdot\frac{1/e^x}{1/e^x} = \lim_{x\to\infty}\frac{1}{\frac{1}{e^x}+1} = \frac{1}{0+1} = 1$$

so H.A.: $Y=1$;

$$\lim_{x\to-\infty}\frac{e^x}{1+e^x} = \frac{e^{-\infty}}{1+e^{-\infty}} = \frac{0}{1+0} = 0$$

so H.A.: $Y=0$