Section 4.2

1.) $f(x) = x^2 + 2x - 1$ on $[0, 1]$;

$f'(x) = 2x + 2$, then

$$f'(c) = \frac{f(1) - f(0)}{1 - 0} = \frac{2 - (-1)}{1} = 3 \rightarrow$$

$$2c + 2 = 3 \rightarrow 2c = 1 \rightarrow c = \tfrac{1}{2}$$

2.) $f(x) = x^{2/3}$ on $[0, 1]$; $f'(x) = \frac{2}{3} x^{-1/3}$,

then

$$f'(c) = \frac{f(1) - f(0)}{1 - 0} = \frac{1 - 0}{1} = 1 \rightarrow$$

$$\frac{2}{3} c^{-1/3} = 1 \rightarrow \frac{2}{c^{1/3}} = 3 \rightarrow c^{1/3} = \frac{2}{3} \rightarrow$$

$$c = \left(\frac{2}{3}\right)^3 \rightarrow c = \frac{8}{27}$$

5.) $f(x) = \arcsin x$ on $[-1, 1]$;

$f'(x) = \dfrac{1}{\sqrt{1 - x^2}}$, then

$$f'(c) = \frac{f(1) - f(-1)}{1 - (-1)} = \frac{\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)}{2} = \frac{\pi}{2} \rightarrow$$

$$\frac{1}{\sqrt{1 - c^2}} = \frac{\pi}{2} \rightarrow \frac{2}{\pi} = \sqrt{1 - c^2} \rightarrow \frac{4}{\pi^2} = 1 - c^2$$

$$\rightarrow c^2 = 1 - \frac{4}{\pi^2} = \frac{\pi^2 - 4}{\pi^2} \rightarrow c = \pm \frac{\sqrt{\pi^2 - 4}}{\pi}$$

6.) $f(x) = \ln(x - 1)$ on $[2, 4]$;

$f'(x) = \dfrac{1}{x - 1}$, then $f'(c) = \dfrac{f(4) - f(2)}{4 - 2} \rightarrow$

$$\frac{1}{c - 1} = \frac{\ln 3 - \cancelto{0}{\ln 1}}{2} \rightarrow 2 = \ln 3 \cdot c - \ln 3 \rightarrow$$

$2 + \ln 3 = \ln 3 \cdot c \to c = \dfrac{2 + \ln 3}{\ln 3}$

9.) $f(x) = x^{2/3}$ on $[-1, 8]$;
let $g(x) = x^2$ and $h(x) = x^{1/3}$ both of which are continuous for all values of x ; and

$f(x) = x^{2/3} = (x^2)^{1/3} = h(x^2) = h(g(x))$

so f is continuous for all values of x (functional composition of continuous functions) ; BUT

$f'(x) = \dfrac{2}{3} x^{-1/3} = \dfrac{2}{3x^{1/3}}$ so f is NOT differentiable at $x = 0$; thus
f is cont. for x in $[-1, 8]$, but is NOT differentiable for x in $(-1, 8)$ so the hypotheses of the MVT are not satisfied.

11.) $f(x) = \sqrt{x - x^2}$ on $[0, 1]$; let
$g(x) = x - x^2$ and $h(x) = \sqrt{x}$ both of which are continuous for x in $[0, 1]$; and

$f(x) = \sqrt{x - x^2} = h(x - x^2) = h(g(x))$ is cont. for x in $[0, 1]$ (functional composition of continuous functions);

and $f'(x) = \dfrac{1}{2}(x - x^2)^{-1/2} \cdot (1 - 2x) = \dfrac{1 - 2x}{2\sqrt{x - x^2}}$

so f is differentiable on $(0,1)$; then

$$f'(c) = \frac{f(1)-f(0)}{1-0} = \frac{0-0}{1} = 0 \rightarrow$$

$$\frac{1-2c}{2\sqrt{c-c^2}} = 0 \rightarrow 1-2c=0 \rightarrow c = \frac{1}{2} .$$

16.)

$$f(x) = \begin{cases} 3 & , \text{ if } x=0 \\ -x^2+3x+a & , \text{ if } 0<x<1 \\ mx+b & , \text{ if } 1 \le x \le 2 \end{cases} ;$$

<u>make f cont. at $x=0$</u> :

$$\lim_{x \to 0^+} (-x^2+3x+a) = 3 \rightarrow \boxed{a=3} ;$$

<u>make f cont. at $x=1$</u> :

$$\lim_{x \to 1^+} (mx+b) = \lim_{x \to 1^-} (-x^2+3x+a) \rightarrow$$

$$m+b = -1+3+3 \rightarrow \boxed{b = 5-m} ;$$

<u>make f diff. at $x=1$</u> :

$$Y = -x^2+3x+a \xrightarrow{D} Y' = -2x+3 \rightarrow Y'(1) = 1 ;$$

$$Y = mx+b \xrightarrow{D} Y' = m \text{ so } \boxed{m=1} \rightarrow$$

$\boxed{b=4}$; thus,

$$f(x) = \begin{cases} -x^2+3x+3 & , \text{ if } 0 \le x < 1 \\ x+4 & , \text{ if } 1 \le x \le 2 . \end{cases}$$

<u>Find values of c</u> :

$$f'(x) = \begin{cases} -2x+3, & \text{if } 0 \le x < 1 \\ 1, & \text{if } 1 \le x \le 2 \end{cases};$$

$$f'(c) = \frac{f(2)-f(0)}{2-0} = \frac{6-3}{2} = \frac{3}{2} \rightarrow$$

<u>case 1</u>: $-2c+3 = 3/2 \rightarrow -2c = -\frac{3}{2} \rightarrow$

$c = \frac{3}{4}$

<u>case 2</u>: $1 = \frac{3}{2}$ (impossible)

34.) $f'(x) = 2x-1 \rightarrow f(x) = x^2 - x + c$
and $x=0,\ y=0 \rightarrow 0 = 0-0+c \rightarrow$
$c=0 \longrightarrow f(x) = x^2 - x$

40.) $g'(x) = \frac{1}{x} + 2x \longrightarrow g(x) = \ln x + x^2 + c$
and $x=1,\ y=-1 \rightarrow -1 = \ln 1 + 1 + c \rightarrow$
$c=-2 \longrightarrow g(x) = \ln x + x^2 - 2$

41.) $f'(x) = e^{2x} \rightarrow f(x) = \frac{1}{2}e^{2x} + c$ and
$x=0,\ y=3/2 \longrightarrow \frac{3}{2} = \frac{1}{2}(1) + c \rightarrow c = 1 \rightarrow$
$f(x) = \frac{1}{2}e^{2x} + 1$

51.) Let $T(t)$ be the temperature (°C) at time t seconds; then by MVT

$$T'(c) = \frac{T(14)-T(0)}{14-0} = \frac{(100)-(-19)}{14} = 8.5\ \frac{°C}{sec.}$$

so at time c, $0 \le c \le 14$, the temperature is increasing at the rate of $8.5\ °C/sec$.

52.) Let $L(t)$ be distance (mi.) traveled after t hours; then by MVT

$$L'(c) = \frac{L(2) - L(0)}{2 - 0} = \frac{159 - 0}{2} = 79.5 \text{ mph}$$

so at time c, $0 \le c \le 2$, speed of truck $L'(c) = 79.5$ mph.

66.) Let $f(x) = \sin x$ on $[a, b]$; by MVT

$$f'(c) = \frac{f(b) - f(a)}{b - a} \longrightarrow$$

$$\cos(c) = \frac{\sin b - \sin a}{b - a} \longrightarrow$$

$$\frac{|\sin b - \sin a|}{|b - a|} = |\cos c| \le 1 \longrightarrow$$

$$|\sin b - \sin a| \le |b - a|.$$