Section 2.2

1.) a.) $\lim_{x \to 1} g(x)$ DNE since

$\lim_{x \to 1^+} g(x) = 0$ and $\lim_{x \to 1^-} g(x) = 1$.

b.) $\lim_{x \to 2} g(x) = 1$

c.) $\lim_{x \to 3} g(x) = 0$

2.) a.) $\lim_{t \to -2} f(t) = 0$

b.) $\lim_{t \to -1} f(t) = -1$

c.) $\lim_{t \to 0} f(t)$ DNE since

$\lim_{t \to 0^+} f(t) = 1$ and $\lim_{t \to 0^-} f(t) = -1$

3.) a.) T

b.) T

c.) F

d.) F

e.) F

f.) T

5.) Recall: $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$;

$$\lim_{x \to 0^+} \frac{x}{|x|} = \lim_{x \to 0^+} \frac{x}{x} = \lim_{x \to 0^+} 1 = 1 ;$$

$$\lim_{x \to 0^-} \frac{x}{|x|} = \lim_{x \to 0^-} \frac{x}{-x} = \lim_{x \to 0^-} -1 = -1 ;$$

so $\lim_{x \to 0} \frac{x}{|x|}$ DNE.

15.) $\lim_{x \to 2} \frac{x+3}{x+6} = \frac{5}{8}$

21.) $\lim_{h \to 0} \frac{3}{\sqrt{3h+1}+1} = \frac{3}{\sqrt{1}+1} = \frac{3}{1+1} = \frac{3}{2}$

22.) $\lim_{h \to 0} \frac{\sqrt{5h+4}-2}{h} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{h \to 0} \frac{\sqrt{5h+4}-2}{h} \cdot \frac{\sqrt{5h+4}+2}{\sqrt{5h+4}+2}$

$= \lim_{h \to 0} \frac{(5h+\cancel{4})-\cancel{4}}{h(\sqrt{5h+4}+2)} = \lim_{h \to 0} \frac{5\cancel{h}}{\cancel{h}(\sqrt{5h+4}+2)}$

$= \frac{5}{2+2} = \frac{5}{4}$

24.) $\lim_{x \to -3} \frac{x+3}{x^2+4x+3} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x \to -3} \frac{\cancel{x+3}}{(\cancel{x+3})(x+1)}$

$= \lim_{x \to -3} \frac{1}{x+1} = \frac{1}{-2} = \frac{-1}{2}$

27.) $\lim_{t \to 1} \frac{t^2+t-2}{t^2-1} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{t \to 1} \frac{(\cancel{t-1})(t+2)}{(\cancel{t-1})(t+1)} = \frac{3}{2}$

30.) $\lim_{y \to 0} \frac{5y^3+8y^2}{3y^4-16y^2} = \lim_{y \to 0} \frac{\cancel{y^2}(5y+8)}{\cancel{y^2}(3y^2-16)}$

$= \frac{8}{-16} = \frac{-1}{2}$

33.) $\lim_{u \to 1} \frac{u^4-1}{u^3-1} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{u \to 1} \frac{(u^2-1)(u^2+1)}{(u-1)(u^2+u+1)}$

$= \lim_{u \to 1} \frac{(\cancel{u-1})(u+1)(u^2+1)}{(\cancel{u-1})(u^2+u+1)} = \frac{(2)(2)}{3} = \frac{4}{3}$

37) $$\lim_{x\to 1} \frac{x-1}{\sqrt{x+3}-2} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 1} \frac{x-1}{\sqrt{x+3}-2} \cdot \frac{\sqrt{x+3}+2}{\sqrt{x+3}+2}$$

$$= \lim_{x\to 1} \frac{(x-1)(\sqrt{x+3}+2)}{(x+3)-4} = \lim_{x\to 1} \frac{\cancel{(x-1)}(\sqrt{x+3}+2)}{\cancel{x-1}}$$

$$= 2+2 = 4$$

40) $$\lim_{x\to -2} \frac{x+2}{\sqrt{x^2+5}-3} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to -2} \frac{x+2}{\sqrt{x^2+5}-3} \cdot \frac{\sqrt{x^2+5}+3}{\sqrt{x^2+5}+3}$$

$$= \lim_{x\to -2} \frac{(x+2)(\sqrt{x^2+5}+3)}{(x^2+5)-9}$$

$$= \lim_{x\to -2} \frac{(x+2)(\sqrt{x^2+5}+3)}{x^2-4}$$

$$= \lim_{x\to -2} \frac{\cancel{(x+2)}(\sqrt{x^2+5}+3)}{\cancel{(x+2)}(x-2)}$$

$$= \frac{3+3}{-4} = -\frac{3}{2}$$

53) a.) $$\lim_{x\to c} f(x)g(x) = \lim_{x\to c} f(x) \cdot \lim_{x\to c} g(x)$$

$$= (5)(-2) = -10$$

b.) $$\lim_{x\to c} 2f(x)g(x) = 2 \cdot \lim_{x\to c} f(x) \cdot \lim_{x\to c} g(x)$$

$$= 2(5)(-2) = -20$$

c.) $$\lim_{x\to c} (f(x)+3g(x))$$

$$= \lim_{x\to c} f(x) + 3 \cdot \lim_{x\to c} g(x) = 5+3(-2) = -1$$

d.) $\lim_{x\to c} \frac{f(x)}{f(x)-g(x)} = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} f(x) - \lim_{x\to c} g(x)}$

$= \frac{5}{5-(-2)} = \frac{5}{7}$

64.) $2 - x^2 \le g(x) \le 2\cos x$, then
$\lim_{x\to 0} (2-x^2) = 2-0 = 2$ and
$\lim_{x\to 0} 2\cos x = 2\cos 0 = 2\cdot(1) = 2$, so
by Squeeze Principle $\lim_{x\to 0} g(x) = 2$

65.) a.) $1 - \frac{x^2}{6} \le \frac{x\sin x}{2-2\cos x} \le 1$, then
$\lim_{x\to 0} \left(1 - \frac{x^2}{6}\right) = 1-0 = 1$ and
$\lim_{x\to 0} 1 = 1$, so by Squeeze Principle
$\lim_{x\to 0} \frac{x\sin x}{2-2\cos x} = 1$.

80.) $\lim_{x\to -2} \frac{f(x)}{x^2} = 1 \rightarrow \frac{\lim_{x\to -2} f(x)}{\lim_{x\to -2} x^2} = 1 \rightarrow$
$\frac{\lim_{x\to -2} f(x)}{4} = 1 \rightarrow \lim_{x\to -2} f(x) = 4$

a.) $\lim_{x\to -2} f(x) = 4$

b.) $\lim_{x\to -2} \frac{f(x)}{x} = \frac{\lim_{x\to -2} f(x)}{\lim_{x\to -2} x} = \frac{4}{-2} = -2$

81.) a.) $\lim_{x\to 2} \frac{f(x)-5}{x-2} = 3 \longrightarrow$

$$\frac{\lim_{x\to 2}(f(x)-5)}{\lim_{x\to 2}(x-2)} = 3 \rightarrow \frac{\lim_{x\to 2}(f(x)-5)}{0} = 3$$

$$\rightarrow \lim_{x\to 2}(f(x)-5) = 0 \rightarrow \lim_{x\to 2} f(x) - \lim_{x\to 2} 5 = 0$$

$$\rightarrow \lim_{x\to 2} f(x) - 5 = 0 \rightarrow \boxed{\lim_{x\to 2} f(x) = 5}.$$

(If $\lim_{x\to 2}(f(x)-5) = k$ $(\neq 5)$, then

$$\frac{\lim_{x\to 2}(f(x)-5)}{0} = \frac{k}{0} = \pm\infty.)$$

82.) $\lim_{x\to 0} \frac{f(x)}{x^2} = 1 \longrightarrow \frac{\lim_{x\to 0} f(x)}{\lim_{x\to 0} x^2} = 1 \longrightarrow$

$$\frac{\lim_{x\to 0} f(x)}{0} = 1 \longrightarrow \lim_{x\to 0} f(x) = 0$$

a.) $\lim_{x\to 0} f(x) = 0$

b.) $\frac{f(x)}{x} = \frac{f(x)\cdot x}{x^2} = \frac{f(x)}{x^2}\cdot x \longrightarrow$

$$\lim_{x\to 0}\frac{f(x)}{x} = \lim_{x\to 0}\frac{f(x)}{x^2}\cdot\lim_{x\to 0} x = (1)\cdot(0) = 0.$$