Section 4.5

7.) $$\lim_{x\to 2} \frac{x-2}{x^2-4} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 2} \frac{1}{2x} = \frac{1}{4}$$

10.) $$\lim_{t\to 1} \frac{t^3-1}{4t^3-t-3} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{t\to 1} \frac{3t^2}{12t^2-1} = \frac{3}{11}$$

13.) $$\lim_{t\to 0} \frac{\sin t^2}{t} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{t\to 0} \frac{\cos t^2 \cdot 2t}{1} = (1)(0) = 0$$

14.) $$\lim_{t\to 0} \frac{\sin 5t}{2t} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{t\to 0} \frac{\cos 5t \cdot 5}{2} = \frac{(1)\cdot 5}{2} = \frac{5}{2}$$

15.) $$\lim_{x\to 0} \frac{8x^2}{\cos x-1} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 0} \frac{16x}{-\sin x}$$

$$\overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 0} \frac{16}{-\cos x} = \frac{16}{-1} = -16$$

18.) $$\lim_{\theta\to -\frac{\pi}{3}} \frac{3\theta+\pi}{\sin(\theta+\frac{\pi}{3})} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{\theta\to -\frac{\pi}{3}} \frac{3}{\cos(\theta+\frac{\pi}{3})} = \frac{3}{1} = 3$$

20.) $$\lim_{x\to 1} \frac{x-1}{\ln x - \sin \pi x} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 1} \frac{1}{\frac{1}{x} - \pi\cos\pi x}$$

$$= \frac{1}{1-\pi(-1)} = \frac{1}{1+\pi}$$

21.) $$\lim_{x\to 0} \frac{x^2}{\ln(\sec x)} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 0} \frac{2x}{\frac{1}{\cancel{\sec x}} \cdot \cancel{\sec x}\tan x}$$

$$= \lim_{x\to 0} \frac{2x}{\tan x} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 0} \frac{2}{\sec^2 x} = \frac{2}{(1)^2} = 2$$

24.) $$\lim_{t\to 0} \frac{t\sin t}{1-\cos t} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{t\to 0} \frac{t\cos t + \sin t}{\sin t}$$

$$\overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{t \to 0} \frac{t \cdot -\sin t + \cos t + \cos t}{\cos t}$$

$$= \frac{0+1+1}{1} = 2$$

26.) $\lim_{x \to \frac{\pi}{2}^-} \left(\frac{\pi}{2} - x\right) \tan x = \text{"}0 \cdot \infty\text{"}$

$$= \lim_{x \to \frac{\pi}{2}^-} \frac{\frac{\pi}{2} - x}{\cot x} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x \to \frac{\pi}{2}^-} \frac{-1}{-\csc^2 x}$$

$$= \lim_{x \to \frac{\pi}{2}^-} \sin^2 x = (1)^2 = 1$$

27.) $\lim_{\theta \to 0} \frac{3^{\sin\theta} - 1}{\theta} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{\theta \to 0} \frac{3^{\sin\theta} \cdot \cos\theta \cdot \ln 3}{1}$

$$= (1)(1) \ln 3 = \ln 3$$

29.) $\lim_{x \to 0} \frac{x \cdot 2^x}{2^x - 1} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x \to 0} \frac{x \cdot 2^x \ln 2 + 2^x}{2^x \ln 2}$

$$= \frac{0+1}{(1)\ln 2} = \frac{1}{\ln 2}$$

32.) $\lim_{x \to \infty} \frac{\log_2 x}{\log_3 (x+3)} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim_{x \to \infty} \frac{\frac{1}{x} \cdot \frac{1}{\ln 2}}{\frac{1}{x+3} \cdot \frac{1}{\ln 3}}$

$$= \lim_{x \to \infty} \frac{x+3}{x} \cdot \frac{\ln 3}{\ln 2} = \lim_{x \to \infty} \left(1 + \frac{3}{x}\right) \cdot \frac{\ln 3}{\ln 2}$$

$$= (1+0) \frac{\ln 3}{\ln 2} = \frac{\ln 3}{\ln 2}$$

34.) $$\lim_{x\to 0^+} \frac{\ln(e^x-1)}{\ln x} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim_{x\to 0^+} \frac{\frac{1}{e^x-1}\cdot e^x}{\frac{1}{x}}$$

$$= \lim_{x\to 0^+} \frac{xe^x}{e^x-1} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 0^+} \frac{xe^x+e^x}{e^x} = \frac{1}{1} = 1$$

35.) $$\lim_{Y\to 0} \frac{\sqrt{5Y+25}-5}{Y} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{Y\to 0} \frac{\frac{1}{2}(5Y+25)^{-1/2}\cdot 5}{1}$$

$$= \lim_{Y\to 0} \frac{5}{2\sqrt{5Y+25}} = \frac{5}{2(5)} = \frac{1}{2}$$

41.) $$\lim_{x\to 1^+} \left(\frac{1}{x-1} - \frac{1}{\ln x}\right) = \text{"}\infty - \infty\text{"}$$

$$= \lim_{x\to 1^+} \frac{\ln x-(x-1)}{(x-1)\ln x} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 1^+} \frac{\frac{1}{x}-1}{(x-1)\cdot\frac{1}{x}+(1)\ln x}\cdot\frac{x}{x}$$

$$= \lim_{x\to 1^+} \frac{1-x}{x-1+x\ln x} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 1^+} \frac{-1}{1+x\cdot\frac{1}{x}+(1)\ln x}$$

$$= \lim_{x\to 1^+} \frac{-1}{1+1+\ln x} = \frac{-1}{2+0} = \frac{-1}{2}$$

44.) $$\lim_{h\to 0} \frac{e^h-1-h}{h^2} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{h\to 0} \frac{e^h-1}{2h}$$

$$\overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{h\to 0} \frac{e^h}{2} = \frac{1}{2}$$

46.) $$\lim_{x\to\infty} \frac{x^2}{e^x} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim_{x\to\infty} \frac{2x}{e^x} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim_{x\to\infty} \frac{2}{e^x}$$

$$= \frac{3}{\infty} = 0$$

53) $\lim_{x\to\infty} (\ln x)^{1/x} = "\infty^0"$

$= \lim_{x\to\infty} e^{\ln(\ln x)^{1/x}} = \lim_{x\to\infty} e^{\frac{1}{x}\ln(\ln x)}$

$= \lim_{x\to\infty} e^{\frac{\ln(\ln x)}{x}} = e^{\lim_{x\to\infty} \frac{\ln(\ln x)}{x}}$

$\overset{"\frac{\infty}{\infty}"}{=} e^{\lim_{x\to\infty} \frac{\frac{1}{\ln x}\cdot\frac{1}{x}}{1}} = e^0 = 1$

56.) $\lim_{x\to\infty} x^{1/\ln x} = "\infty^0"$

$= \lim_{x\to\infty} e^{\ln x^{1/\ln x}} = \lim_{x\to\infty} e^{\frac{1}{\ln x}\cdot \ln x}$

$= \lim_{x\to\infty} e^1 = e$

57.) $\lim_{x\to\infty} (1+2x)^{\frac{1}{2\ln x}} = "\infty^0"$

$= \lim_{x\to\infty} e^{\ln(1+2x)^{1/2\ln x}} = \lim_{x\to\infty} e^{\frac{1}{2\ln x}\cdot\ln(1+2x)}$

$= e^{\lim_{x\to\infty} \frac{\ln(1+2x)}{2\ln x}} \overset{"\frac{\infty}{\infty}"}{=} e^{\lim_{x\to\infty} \frac{\frac{1}{1+2x}\cdot \cancel{2}}{\cancel{2}\cdot 1/x}}$

$= e^{\lim_{x\to\infty} \frac{x}{1+2x}} \overset{"\frac{\infty}{\infty}"}{=} e^{\lim_{x\to\infty} \frac{1}{2}} = e^{1/2}$

58.) $\lim_{x\to 0} (e^x+x)^{1/x} = "1^{\pm\infty}"$

$$= \lim_{x\to 0} e^{\ln(e^x+x)^{1/x}} = \lim_{x\to 0} e^{\frac{1}{x}\ln(e^x+x)}$$

$$= e^{\lim_{x\to 0} \frac{\ln(e^x+x)}{x}} \overset{\text{"}\frac{0}{0}\text{"}}{=} e^{\lim_{x\to 0} \frac{\frac{1}{e^x+x}\cdot(e^x+1)}{1}}$$

$$= e^{\frac{2}{1+0}} = e^2$$

59) $$\lim_{x\to 0^+} x^x = \text{"}0^0\text{"} = \lim_{x\to 0^+} e^{\ln x^x}$$

$$= \lim_{x\to 0^+} e^{x\ln x} \overset{\text{"}0\cdot\infty\text{"}}{=} e^{\lim_{x\to 0^+} \frac{\ln x}{1/x}}$$

$$\overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} e^{\lim_{x\to 0^+} \frac{\frac{1}{x}}{-1/x^2}} = e^{\lim_{x\to 0^+}(-x)} = e^0 = 1$$

60) $$\lim_{x\to 0^+} \left(1+\frac{1}{x}\right)^x = \text{"}\infty^0\text{"}$$

$$= \lim_{x\to 0^+} e^{\ln\left(1+\frac{1}{x}\right)^x} = \lim_{x\to 0^+} e^{x\ln(1+1/x)}$$

$$\overset{\text{"}0\cdot\infty\text{"}}{=} \lim_{x\to 0^+} e^{\frac{\ln(1+1/x)}{1/x}} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} e^{\lim_{x\to 0^+} \frac{\frac{1}{1+1/x}\cdot\cancel{\frac{-1}{x^2}}}{\cancel{-1/x^2}}}$$

$$= e^{\frac{1}{\infty}} = e^0 = 1$$

79.) $f(x) = \begin{cases} \dfrac{9x - 3\sin 3x}{5x^3} & , \text{ if } x \neq 0 \\ c & , \text{ if } x = 0 \end{cases}$

$$\lim_{x \to 0} \frac{9x - 3\sin 3x}{5x^3} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x \to 0} \frac{9 - 3 \cdot \cos 3x \cdot 3}{15x^2}$$

$$= \lim_{x \to 0} \frac{9 - 9\cos 3x}{15x^2} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x \to 0} \frac{-9 \cdot -\sin 3x \cdot 3}{30x}$$

$$= \lim_{x \to 0} \frac{27 \sin 3x}{30x} = \lim_{x \to 0} \frac{9 \sin 3x}{10x}$$

$$\overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x \to 0} \frac{9 \cdot \cos 3x \cdot 3}{10} = \frac{27(1)}{10} = \frac{27}{10} ;$$

let $c = \frac{27}{10}$; then

i.) $f(0) = \frac{27}{10}$

ii.) $\lim_{x \to 0} f(x) = \frac{27}{10}$

iii.) $\lim_{x \to 0} f(x) = f(0)$

and f is continuous at $x = 0$.