Section 2.4

2.) a.) T
b.) F
c.) F
d.) T
e.) T
f.) T
g.) T
h.) T
i.) T

3.) a.) $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} \left(\frac{x}{2}+1\right) = 1+1 = 2$,

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (3-x) = 3-2 = 1$

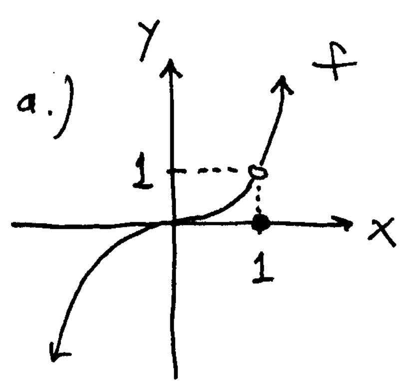
b.) $\lim_{x\to 2} f(x)$ DNE because of a.)

c.) $\lim_{x\to 4^-} f(x) = \lim_{x\to 4^-} \left(\frac{x}{2}+1\right) = 2+1 = 3$,

$\lim_{x\to 4^+} f(x) = \lim_{x\to 4^+} \left(\frac{x}{2}+1\right) = 2+1 = 3$

d.) $\lim_{x\to 4} f(x) = 3$ because of c.)

7.) a.)



$$f(x) = \begin{cases} x^3, & x \neq 1 \\ 0, & x = 1 \end{cases}$$

b.) $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} x^3 = 1^3 = 1$,

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} x^3 = 1^3 = 1$

c.) $\lim_{x \to 1} f(x) = 1$ because of b.)

12.) $\lim_{x \to 1^+} \sqrt{\frac{x-1}{x+2}} = \sqrt{\frac{1-1}{1+3}} = \sqrt{\frac{0}{4}} = \sqrt{0} = 0$

15.) $\lim_{h \to 0^+} \frac{\sqrt{h^2+4h+5} - \sqrt{5}}{h}$

$$\overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{h \to 0^+} \frac{\sqrt{h^2+4h+5} - \sqrt{5}}{h} \cdot \frac{\sqrt{h^2+4h+5} + \sqrt{5}}{\sqrt{h^2+4h+5} + \sqrt{5}}$$

$$= \lim_{h \to 0^+} \frac{(h^2+4h+\cancel{5}) - \cancel{5}}{h\left(\sqrt{h^2+4h+5} + \sqrt{5}\right)}$$

$$= \lim_{h \to 0^+} \frac{\cancel{h}(h+4)}{\cancel{h}\left(\sqrt{h^2+4h+5} + \sqrt{5}\right)} = \frac{4}{\sqrt{5}+\sqrt{5}}$$

$$= \frac{4}{2\sqrt{5}} = \frac{2}{\sqrt{5}}$$

17.) b.) $\lim_{x \to -2^-} (x+3) \cdot \frac{|x+2|}{x+2}$

$x = -2$

$x + 2 < 0$

$$= \lim_{x \to -2^-} (x+3) \cdot \frac{-\cancel{(x+2)}}{\cancel{x+2}}$$

$$= \lim_{x \to -2^-} -(x+3) = -(-2+3) = -1$$

23.) $\lim_{y \to 0} \frac{\sin 3y}{4y} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{y \to 0} \left(\frac{\sin 3y}{3y}\right) \cdot \frac{3}{4}$

$= (1) \cdot \frac{3}{4} = 3/4$

24.) $\lim\limits_{h\to 0^-} \dfrac{h}{\sin 3h} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{h\to 0^-} \dfrac{1}{3}\cdot\dfrac{3h}{\sin 3h} = \dfrac{1}{3}(1) = \dfrac{1}{3}$

26.) $\lim\limits_{t\to 0} \dfrac{2t}{\tan t} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{t\to 0} \dfrac{2t}{\frac{\sin t}{\cos t}}$

$= \lim\limits_{t\to 0} 2\cdot\dfrac{t}{\sin t}\cdot\cos t = 2(1)(\cos 0)$

$= 2(1)(1) = 2$

27.) $\lim\limits_{x\to 0} \dfrac{x\csc 2x}{\cos 5x} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{x\to 0} \dfrac{x}{\cos 5x}\cdot\dfrac{1}{\sin 2x}$

$= \lim\limits_{x\to 0} \dfrac{1}{2}\cdot\dfrac{2x}{\sin 2x}\cdot\dfrac{1}{\cos 5x} = \dfrac{1}{2}(1)\left(\dfrac{1}{1}\right) = \dfrac{1}{2}$

29.) $\lim\limits_{x\to 0} \dfrac{x + x\cos x}{\sin x\cos x} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{x\to 0} \left(\dfrac{x}{\sin x}\cdot\dfrac{1+\cos x}{\cos x}\right)$

$= (1)\cdot\dfrac{1+1}{1} = 2$

34.) $\lim\limits_{h\to 0} \dfrac{\sin(\sin h)}{\sin h} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{\substack{k\to 0\\ k=\sin h}} \dfrac{\sin k}{k} = 1$

35.) $\lim\limits_{\theta\to 0} \dfrac{\sin\theta}{\sin 2\theta} = \lim\limits_{\theta\to 0} \dfrac{\cancel{\sin\theta}}{2\cancel{\sin\theta}\cos\theta}$

$= \dfrac{1}{2(1)} = \dfrac{1}{2}$

36.) $\lim\limits_{x\to 0} \dfrac{\sin 5x}{\sin 4x} = \lim\limits_{x\to 0}\left(\dfrac{5}{4}\cdot\dfrac{\sin 5x}{5x}\cdot\dfrac{4x}{\sin 4x}\right)$

$= \dfrac{5}{4}\cdot(1)\cdot(1) = \dfrac{5}{4}$