Section 2.6

9.) $-1 \le \sin 2x \le +1 \longrightarrow \frac{-1}{x} \le \frac{\sin 2x}{x} \le \frac{1}{x}$

and $\lim_{x\to\infty} \frac{-1}{x} = \text{"}\frac{-1}{\infty}\text{"} = 0$,

$\lim_{x\to\infty} \frac{1}{x} = \text{"}\frac{1}{\infty}\text{"} = 0$, so by Squeeze Principle $\lim_{x\to\infty} \frac{\sin 2x}{x} = 0$.

11.) $-1 \le \sin t \le +1 \longrightarrow \frac{-1}{t} \le \frac{\sin t}{t} \le \frac{1}{t}$

and $\lim_{t\to-\infty} \frac{-1}{t} = \text{"}\frac{-1}{-\infty}\text{"} = 0$,

$\lim_{t\to-\infty} \frac{1}{t} = \text{"}\frac{1}{-\infty}\text{"} = 0$, so by Squeeze Principle $\lim_{t\to-\infty} \frac{\sin t}{t} = 0$; AND

$-1 \le \cos t \le +1 \longrightarrow \frac{-1}{t} \le \frac{\cos t}{t} \le \frac{1}{t}$, and

$\lim_{t\to-\infty} \frac{-1}{t} = 0 = \lim_{t\to-\infty} \frac{1}{t}$, so by Squeeze Principle $\lim_{t\to-\infty} \frac{\cos t}{t} = 0$;

NOW

$$\lim_{t\to-\infty} \frac{2-t+\sin t}{t+\cos t} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim_{t\to-\infty} \frac{2-t+\sin t}{t+\cos t}\cdot\frac{\frac{1}{t}}{\frac{1}{t}}$$

$$= \lim_{t\to-\infty} \frac{\frac{2}{t}-1+\frac{\sin t}{t}}{1+\frac{\cos t}{t}} = \frac{(0)-1+(0)}{1+(0)} = -1$$

16.) $\lim_{x\to\pm\infty} \frac{3x+7}{x^2-2} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim_{x\to\pm\infty} \frac{3x+7}{x^2-2} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$

$= \lim_{x\to\pm\infty} \frac{\frac{3}{x}+\frac{7}{x^2}}{1-\frac{2}{x^2}} = \frac{(0)+(0)}{1-(0)} = 0$

20.) $\lim_{x\to\pm\infty} \frac{9x^4+x}{2x^4+5x^2-x+6} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim_{x\to\pm\infty} \frac{9x^4+x}{2x^4+5x^2-x+6} \cdot \frac{\frac{1}{x^4}}{\frac{1}{x^4}}$

$= \lim_{x\to\pm\infty} \frac{9+\frac{1}{x^3}}{2+\frac{5}{x^2}-\frac{1}{x^3}+\frac{6}{x^4}} = \frac{9+(0)}{2+(0)-(0)+(0)} = \frac{9}{2}$

29.) $\lim_{x\to-\infty} \frac{x^{1/3}-x^{1/5}}{x^{1/3}+x^{1/5}} = \text{"}\frac{\infty-\infty}{\infty-\infty}\text{"}$

$= \lim_{x\to-\infty} \frac{x^{1/3}-x^{1/5}}{x^{1/3}+x^{1/5}} \cdot \frac{\frac{1}{x^{1/3}}}{\frac{1}{x^{1/3}}} = \lim_{x\to-\infty} \frac{1-x^{1/5-1/3}}{1+x^{1/5-1/3}}$

$= \lim_{x\to-\infty} \frac{1-\frac{1}{x^{2/15}}}{1+\frac{1}{x^{2/15}}} = \frac{1-(0)}{1+(0)} = 1$

30.) $\lim_{x\to\infty} \frac{x^{-1}+x^{-4}}{x^{-2}+x^{-3}} = \lim_{x\to\infty} \frac{\frac{1}{x}+\frac{1}{x^4}}{\frac{1}{x^2}+\frac{1}{x^3}} \cdot \frac{x^2}{x^2}$

$= \lim_{x\to\infty} \frac{x+\frac{1}{x^2}}{1+\frac{1}{x}} = \frac{\infty+0}{1+0} = \infty$

37.) $\lim\limits_{x \to 0^+} \dfrac{1}{3x} = \text{"}\dfrac{1}{0^+}\text{"} = +\infty$

39.) $\lim\limits_{x \to 2^-} \dfrac{3}{x-2} = \text{"}\dfrac{3}{0^-}\text{"} = -\infty$ $\qquad x = 2$

42.) $\lim\limits_{x \to -5^-} \dfrac{3x}{2x+10} = \text{"}\dfrac{-15}{0^+}\text{"} = -\infty$ $\qquad x = -5$

44.) $\lim\limits_{x \to 0} \dfrac{-1}{x^2(x+1)} = \text{"}\dfrac{-1}{(0^+)(1)}\text{"} = \text{"}\dfrac{-1}{0^+}\text{"} = -\infty$

49.) $\lim\limits_{x \to \frac{\pi}{2}^-} \tan x = \lim\limits_{x \to \frac{\pi}{2}^-} \dfrac{\sin x}{\cos x}$ $\qquad x = \frac{\pi}{2}$

$= \dfrac{\sin \frac{\pi}{2}}{\cos \frac{\pi}{2}} = \text{"}\dfrac{1}{0^+}\text{"} = +\infty$

50.) $\lim\limits_{x \to -\frac{\pi}{2}^+} \sec x = \lim\limits_{x \to -\frac{\pi}{2}^+} \dfrac{1}{\cos x}$ $\qquad x = -\frac{\pi}{2}^+$

$= \dfrac{1}{\cos \frac{\pi}{2}} = \text{"}\dfrac{1}{0^+}\text{"} = +\infty$

54.) a.) $\lim\limits_{x \to 1^+} \dfrac{x}{x^2-1} = \text{"}\dfrac{1}{0^+}\text{"} = +\infty$ $\qquad x = 1$

b.) $\lim\limits_{x \to 1^-} \dfrac{x}{x^2-1} = \text{"}\dfrac{1}{0^-}\text{"} = -\infty$

$x = -1$

c.) $\lim\limits_{x \to -1^+} \dfrac{x}{x^2-1} = \text{"}\dfrac{-1}{0^-}\text{"} = +\infty$

d.) $\lim\limits_{x \to -1^-} \dfrac{x}{x^2-1} = \text{"}\dfrac{-1}{0^+}\text{"} = -\infty$

57.) a.) $\displaystyle \lim_{x\to 2^+} \frac{x^2-3x+2}{x^3-4x} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 2^+} \frac{\cancel{(x-2)}(x-1)}{x\,\cancel{(x-2)}(x+2)}$

$\displaystyle = \frac{1}{2\cdot 4} = \frac{1}{8}$

b.) $\displaystyle \lim_{x\to -2^+} \frac{x^2-3x+2}{x^3-4x} = \lim_{x\to -2^+} \frac{x-1}{x(x+2)}$

$\displaystyle = \text{"}\frac{-3}{(-2)(0^+)}\text{"} = \text{"}\frac{-3}{0^-}\text{"} = +\infty$ $x=-2$

c.) $\displaystyle \lim_{x\to 0^-} \frac{x^2-3x+2}{x^3-4x} = \lim_{x\to 0^-} \frac{x-1}{x(x+2)}$

$\displaystyle = \frac{-1}{(0^-)(2)} = \text{"}\frac{-1}{0^-}\text{"} = +\infty$ $x=0$

d.) $\displaystyle \lim_{x\to 1^+} \frac{x^2-3x+2}{x^3-4x} = \lim_{x\to 1^+} \frac{x-1}{x(x+2)}$

$\displaystyle = \frac{0}{3} = 0$

e.) $\displaystyle \lim_{x\to 0^+} \frac{x^2-3x+2}{x^3-4x} = \lim_{x\to 0^+} \frac{x-1}{x(x+2)}$

$\displaystyle = \text{"}\frac{-1}{(0^+)(2)}\text{"} = \text{"}\frac{-1}{0^+}\text{"} = -\infty$ $x=0$

so (because of c.)

$\displaystyle \lim_{x\to 0} \frac{x^2-3x+2}{x^3-4x}$ DNE

59.) a.) $\displaystyle \lim_{t\to 0^+} \left(2-\frac{3}{t^{1/3}}\right) = 2-\left(\frac{3}{0^+}\right) = 2-\infty$

$= -\infty$

b.) $\lim\limits_{t \to 0^-} \left(2 - \frac{3}{t^{1/3}}\right) = 2 - \left(\frac{3}{0^-}\right) = 2 - (-\infty)$

$= 2 + \infty = \infty$

62.) a.) $\lim\limits_{x \to 0^+} \left(\frac{1}{x^{1/3}} - \frac{1}{(x-1)^{4/3}}\right) = \text{"}\left(\frac{1}{0^+}\right)\text{"} - \frac{1}{1}$

$= \infty - 1 = \infty$

b.) $\lim\limits_{x \to 0^-} \left(\frac{1}{x^{1/3}} - \frac{1}{(x-1)^{4/3}}\right) = \left(\text{"}\frac{1}{0^-}\text{"}\right) - 1 = -\infty - 1$

$= -\infty$

c.) $\lim\limits_{x \to 1^+} \left(\frac{1}{x^{1/3}} - \frac{1}{(x-1)^{4/3}}\right) = 1 - \left(\text{"}\frac{1}{0^+}\text{"}\right)$

$= 1 - \infty = -\infty$

d.) $\lim\limits_{x \to 1^-} \left(\frac{1}{x^{1/3}} - \frac{1}{(x-1)^{4/3}}\right) = 1 - \left(\text{"}\frac{1}{0^+}\text{"}\right)$

$= 1 - \infty = -\infty$

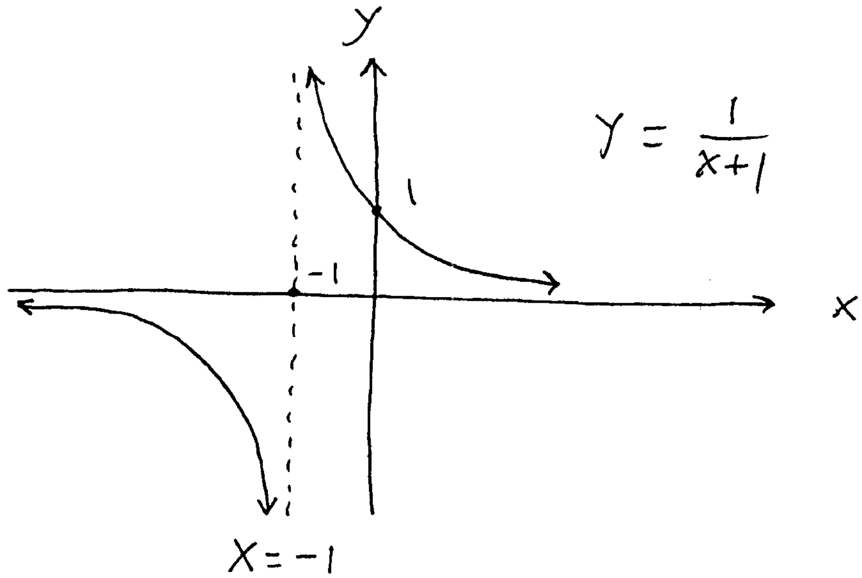
64.) $y = \frac{1}{x+1}$; $x=0$: $\underline{y=1}$

$y = 0$ (impossible) ;

$\lim\limits_{x \to \pm\infty} \frac{1}{x+1} = \text{"}\frac{1}{\pm\infty}\text{"} = 0$ so $\boxed{\text{H.A.} : y=0}$;

$\lim\limits_{x \to -1^+} \frac{1}{x+1} = \text{"}\frac{1}{0^+}\text{"} = +\infty$, $x=-1$

$\lim\limits_{x \to -1^-} \frac{1}{x+1} = \text{"}\frac{1}{0^-}\text{"} = -\infty$, so $\boxed{\text{V.A.} : x=-1}$;



67) $y = \dfrac{x+3}{x+2}$; $\quad x = 0 : \; \underline{y = 3/2}$,

$y = 0 : \; \dfrac{x+3}{x+2} = 0 \rightarrow x+3 = 0 \rightarrow \underline{x = -3}$;

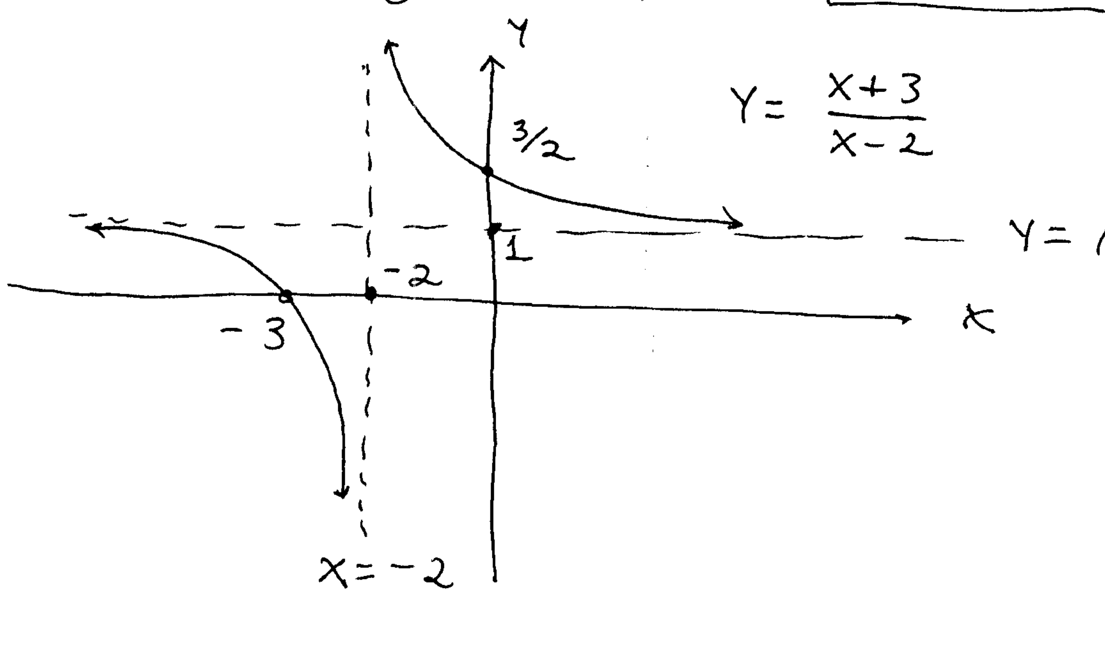
$$\lim_{x\to\pm\infty} \frac{x+3}{x+2} = \lim_{x\to\pm\infty} \frac{x+3}{x+2}\cdot\frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to\pm\infty} \frac{1+3/x}{1+2/x}$$

$$= \frac{1+0}{1+0} = 1 \text{ so } \boxed{\text{H.A.}: \; y = 1} \; ;$$

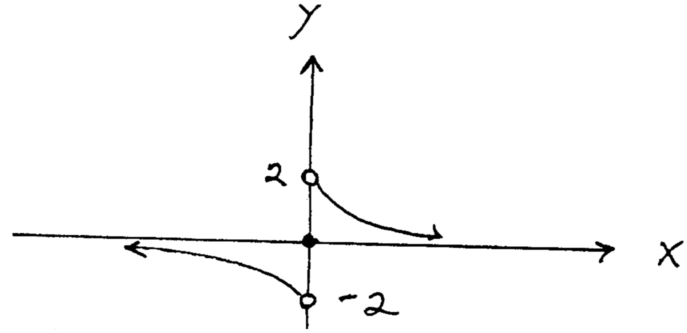
$$\lim_{x\to -2^+} \frac{x+3}{x+2} = \text{"}\frac{1}{0^+}\text{"} = +\infty \; ,$$

$$\lim_{x\to -2^-} \frac{x+3}{x+2} = \text{"}\frac{1}{0^-}\text{"} - \infty \; , \text{ so } \boxed{\text{V.A.}: \; x = -2} \; ;$$

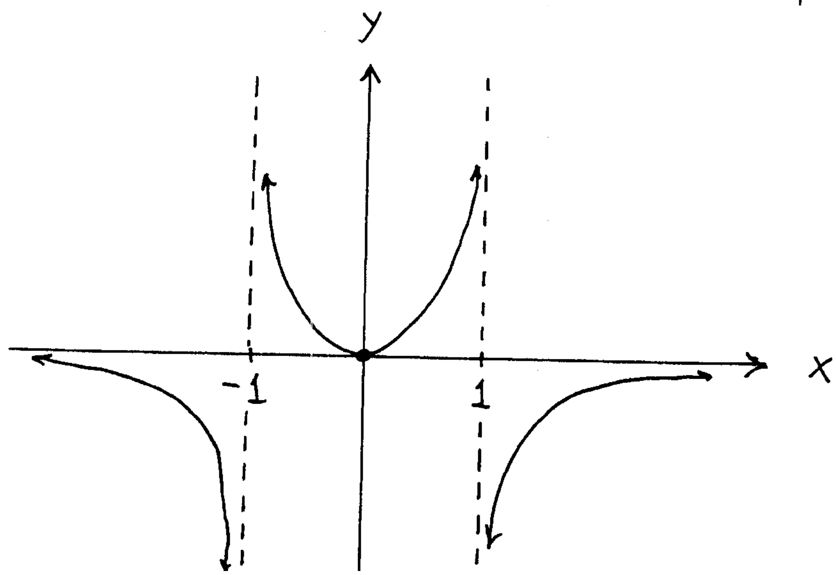
$x = -2$



70.)



71.)



86.) $\lim\limits_{x\to\infty} \left(\sqrt{x^2+x} - \sqrt{x^2-x}\right) = \text{"}\infty - \infty\text{"}$

$= \lim\limits_{x\to\infty} \left(\sqrt{x^2+x} - \sqrt{x^2-x}\right) \dfrac{\left(\sqrt{x^2+x} + \sqrt{x^2-x}\right)}{\left(\sqrt{x^2+x} + \sqrt{x^2-x}\right)}$

$= \lim\limits_{x\to\infty} \dfrac{(\cancel{x^2}+x) - (\cancel{x^2}-x)}{\sqrt{x^2+x} + \sqrt{x^2-x}}$

$= \lim\limits_{x\to\infty} \dfrac{2x}{\sqrt{x^2(1+1/x)} + \sqrt{x^2(1-1/x)}}$

$= \lim\limits_{x\to\infty} \dfrac{2x}{\sqrt{x^2}\sqrt{1+1/x} + \sqrt{x^2}\sqrt{1-1/x}}$

$= \lim\limits_{\substack{x\to\infty \\ (x>0)}} \dfrac{2x}{|x|\sqrt{1+1/x} + |x|\sqrt{1-\frac{1}{x}}}$

$= \lim\limits_{x\to\infty} \dfrac{2x}{x\sqrt{1+\frac{1}{x}} + x\sqrt{1-\frac{1}{x}}} \cdot \dfrac{\frac{1}{x}}{\frac{1}{x}} = \lim\limits_{x\to\infty} \dfrac{2}{\sqrt{1+\frac{1}{x}} + \sqrt{1-\frac{1}{x}}}$

$= \dfrac{2}{1+1} = \dfrac{2}{2} = 1 .$

99.) $Y = \dfrac{x^2+1}{x-1}$; $x=0$: $\underline{Y=-1}$,

$Y=0$: $\dfrac{x^2+1}{x-1} = 0 \rightarrow$

$x^2+1=0$ (impossible) ;

$\lim\limits_{x\to\infty} \dfrac{x^2+1}{x-1} \cdot \dfrac{\frac{1}{x}}{\frac{1}{x}} = \lim\limits_{x\to\infty} \dfrac{x+1/x}{1-1/x} = \dfrac{\infty+0}{1-0} = \infty$;

$\lim\limits_{x\to-\infty} \dfrac{x^2+1}{x-1} \cdot \dfrac{1/x}{1/x} = \lim\limits_{x\to-\infty} \dfrac{x+1/x}{1-1/x} = \dfrac{-\infty+0}{1-0} = -\infty$;

$\boxed{\text{No H.A.}}$

$\lim\limits_{x\to1^+} \dfrac{x^2+1}{x-1} = \text{"}\dfrac{2}{0^+}\text{"} = +\infty$,

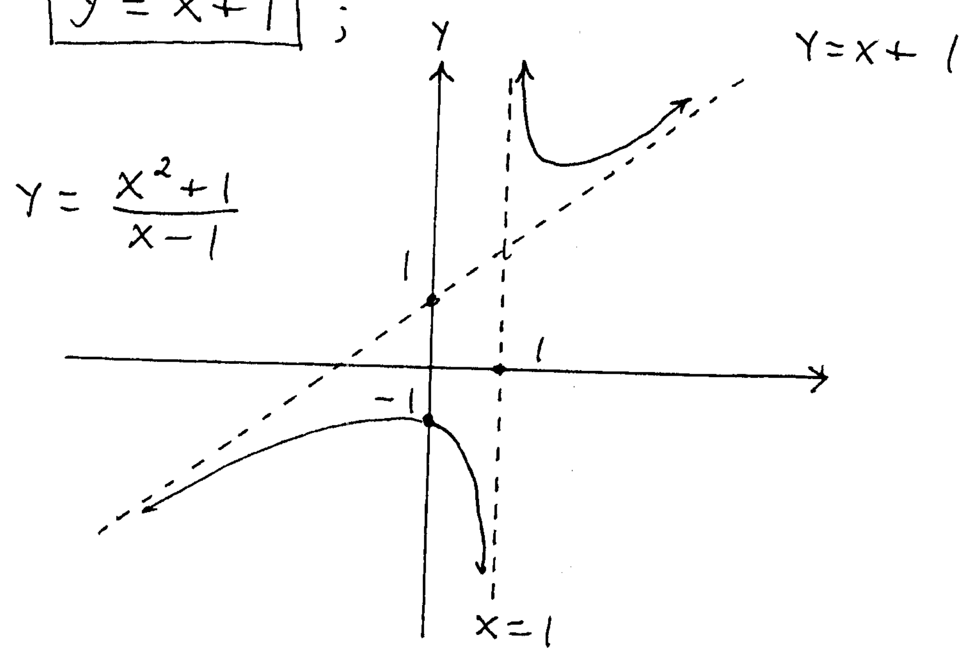
$\lim\limits_{x\to1^-} \dfrac{x^2+1}{x-1} = \text{"}\dfrac{2}{0^-}\text{"} = -\infty$, so

$\boxed{\text{V.A. : } x=1}$;

$$\begin{array}{r} x+1 \\ x-1 \overline{\smash{)}\, x^2+1} \\ -(x^2-x) \\ \hline x+1 \\ -(x-1) \\ \hline 2 \end{array}$$

$Y = \dfrac{x^2+1}{x-1} = x+1+\dfrac{2}{x-1}$

so Tilted asymptote is $\boxed{y=x+1}$;



104.) $Y = \frac{x^3+1}{x^2}$; $x=0$ (impossible) ;

$Y=0$: $\frac{x^3+1}{x^2} = 0 \rightarrow x^3+1=0$

$\rightarrow x=-1$;

$\lim_{x\to\infty} \frac{x^3+1}{x^2} = \lim_{x\to\infty}\left(x+\frac{1}{x^2}\right) = \infty + 0 = \infty$;

$\lim_{x\to-\infty} \frac{x^3+1}{x^2} = \lim_{x\to-\infty}\left(x+\frac{1}{x^2}\right) = -\infty + 0 = -\infty$,

so $\boxed{\text{No H.A.}}$;

$\lim_{x\to 0^+} \frac{x^3+1}{x^2} = \text{"}\frac{1}{0^+}\text{"} = +\infty$,

$\lim_{x\to 0^-} \frac{x^3+1}{x^2} = \text{"}\frac{1}{0^+}\text{"} = +\infty$, so $\boxed{\text{V.A.} : x=0}$;

$Y = \frac{x^3+1}{x^2} = x + \frac{1}{x^2}$, so Tilted asymptote

is $\boxed{Y=X}$;