Section 3.3

2.) $y = x^2 + x + 8 \xrightarrow{D} y' = 2x + 1$
$\xrightarrow{D} y'' = 2$

7.) $w = 3z^{-2} - \frac{1}{z} = 3 \cdot z^{-2} - z^{-1} \xrightarrow{D}$
$w' = -6z^{-3} + z^{-2} \xrightarrow{D} w'' = 18z^{-4} - 2z^{-3}$

8.) $s = -2t^{-1} + \frac{4}{t^2} = -2t^{-1} + 4t^{-2} \xrightarrow{D}$
$s' = 2t^{-2} - 8t^{-3} \xrightarrow{D} s'' = -4t^{-3} + 24t^{-4}$

17.) $y = \frac{2x+5}{3x-2} \xrightarrow{D}$

$$y' = \frac{(3x-2)(2) - (2x+5)(3)}{(3x-2)^2}$$

20.) $f(t) = \frac{t^2-1}{t^2+t-2} \xrightarrow{D}$

$$f'(t) = \frac{(t^2+t-2)(2t) - (t^2-1)(2t+1)}{(t^2+t-2)^2}$$

$y = x^3 e^x \xrightarrow{D}$
$y' = x^3 e^x + 3x^2 e^x$

$y = \frac{1}{120} x^5 \xrightarrow{D} y' = \frac{1}{120} \cdot 5x^4 = \frac{1}{24} x^4$
$\xrightarrow{D} y'' = \frac{1}{24} \cdot 4x^3 = \frac{1}{6} x^3 \xrightarrow{D}$

$$y''' = \frac{1}{6}\cdot 3x^2 = \frac{1}{2}x^2 \xrightarrow{D}$$

$$y^{(4)} = \frac{1}{2}\cdot 2x = x \xrightarrow{D} \quad y^{(5)} = 1 \xrightarrow{D}$$

$$y^{(6)} = y^{(7)} = y^{(8)} = \cdots = 0$$

46.) $s = \dfrac{t^2+5t-1}{t^2} = 1 + 5t^{-1} - t^{-2} \xrightarrow{D}$

$$s' = 0 - 5t^{-2} + 2t^{-3} \xrightarrow{D}$$

$$s'' = 10t^{-3} - 6t^{-4}$$

51.) $w = 3z^2 e^z \xrightarrow{D}$

$$w' = 3z^2\cdot e^z + 6z\cdot e^z \xrightarrow{D}$$

$$w'' = (3z^2\cdot e^z + 6z\cdot e^z) + (6z\cdot e^z + 6e^z)$$

52.) $w = e^z(z-1)(z^2+1) \xrightarrow{D}$ (triple product rule)

$$w' = e^z\cdot(z-1)(z^2+1) + e^z(1)(z^2+1) + e^z(z-1)(2z)$$

$$= e^z\left[z^3 - \cancel{z^2} + z - \cancel{1} + \cancel{z^2} + \cancel{1} + 2z^2 - 2z\right]$$

$$= e^z\left[z^3 + 2z^2 - z\right] \xrightarrow{D}$$

$$w'' = e^z\cdot(3z^2 + 4z - 1) + e^z(z^3 + 2z^2 - z)$$

55.) a.) $y = x^3 - 4x + 1$ at $(2,1)$ so $y' = 3x^2 - 4$ and <u>slope</u> of <u>tangent</u> line at $(2,1)$ is

$Y' = 3(2)^2 - 4 = 8$; then slope of $\perp$ line is

$m = -1/8$ and equation of $\perp$ line at $(2,1)$ is

$$Y - 1 = \frac{-1}{8}(X-2) \rightarrow Y = \frac{-1}{8}X + \frac{5}{4}$$

57.) $Y = \dfrac{4X}{X^2+1} \xrightarrow{D} Y' = \dfrac{(X^2+1)(4) - 4X(2X)}{(X^2+1)^2}$

$$= \frac{4X^2+4-8X^2}{(X^2+1)^2} = \frac{4-4X^2}{(X^2+1)^2}$$

a.) For $(0,0)$ slope of tangent line is $m = Y' = \dfrac{4}{(1)^2} = 4$, so equation of line is

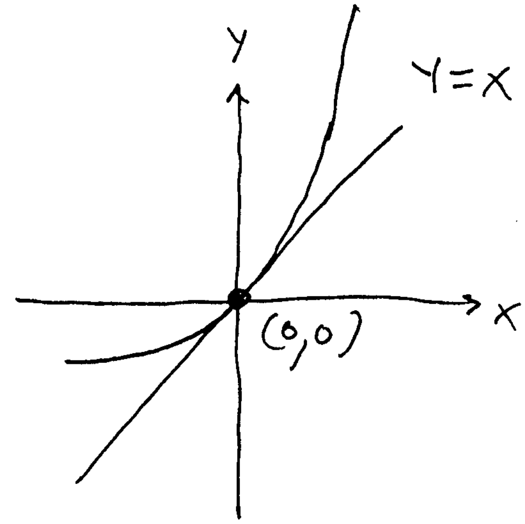
$$Y - 0 = 4(X-0) \rightarrow Y = 4X .$$

b.) For $(1,2)$ slope of tangent line is $m = Y' = \dfrac{0}{(1)^2} = 0$, so equation of line is

$$Y - 2 = 0(X-1) \rightarrow Y = 2 .$$

59.) $Y = AX^2 + BX + C$ passes through point $\boxed{(1,2)}$; is

$Y = Ax^2 + Bx + C$



~~tangent to~~ $Y = X$ at $(0,0)$ so also passes through $\boxed{(0,0)}$; thus

$Y' = 2AX + B$ and $Y' = 1$ have the same value when $X = 0$, i.e.,

$2A(0) + B = 1 \longrightarrow \boxed{B = 1}$;

thus $Y = AX^2 + X + C$

$(X=1, Y=2) \quad 2 = A + 1 + C \longrightarrow \boxed{A + C = 1}$;

$(X=0, Y=0) \quad 0 = 0 + 0 + C \longrightarrow \boxed{C = 0}$

so $\boxed{A=1}$ and $\boxed{Y = X^2 + X}$.

66.) a.) $Y = X^3 - 6X^2 + 5X \xrightarrow{D}$

$Y' = 3X^2 - 12X + 5$; at $(0,0)$ slope of tangent line is

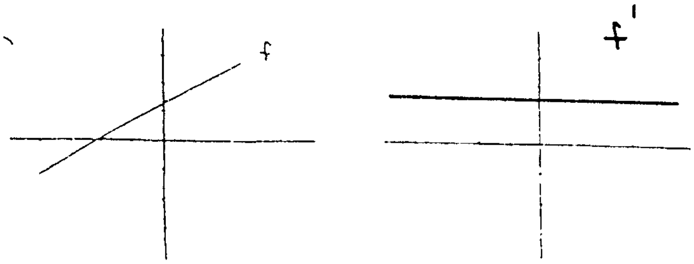
$m = Y' = 0 - 0 + 5 = 5$ so equation of line is

$Y - 0 = 5(X - 0) \longrightarrow Y = 5X$

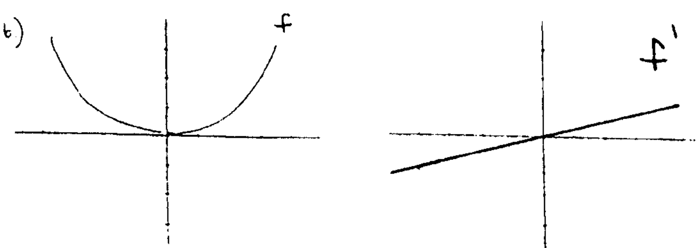
Worksheet 4

1. Use the given graph of function f to sketch a graph of its derivative, f'.

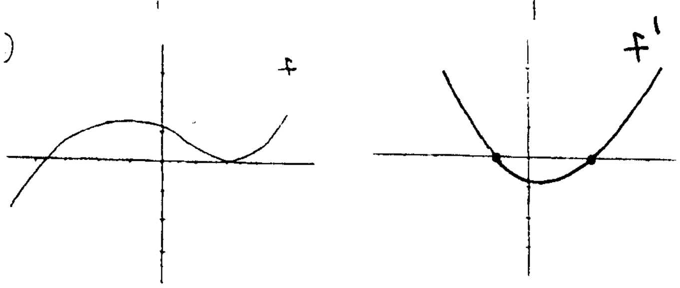
a)



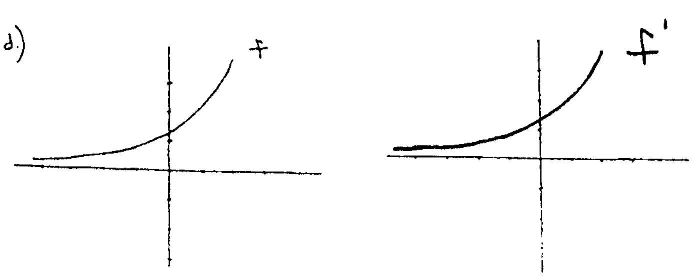
b)



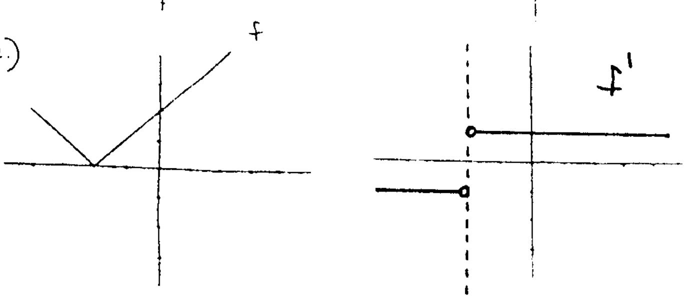
c)



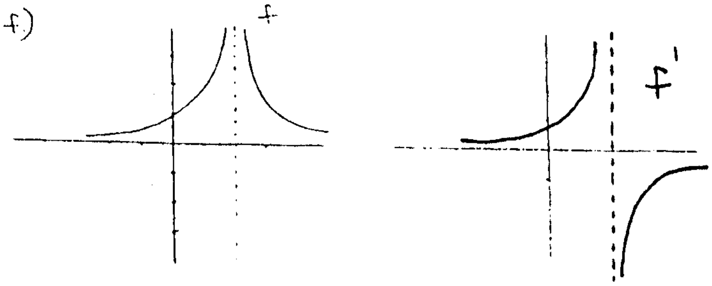
d.)



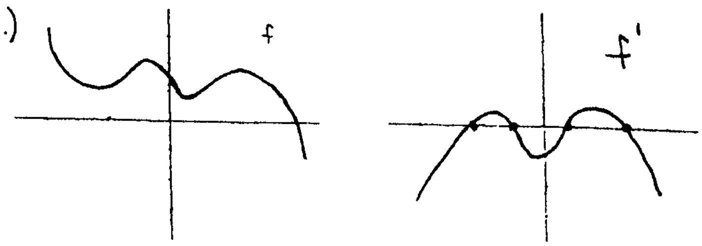
e.)



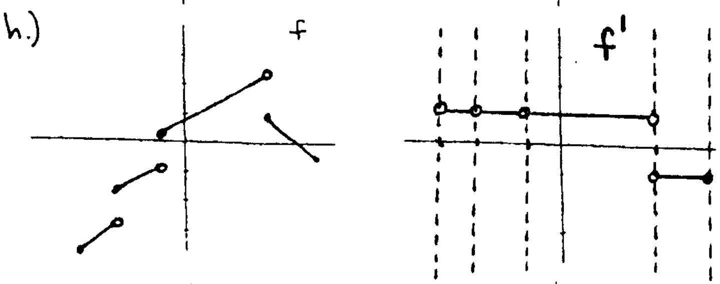
f.)



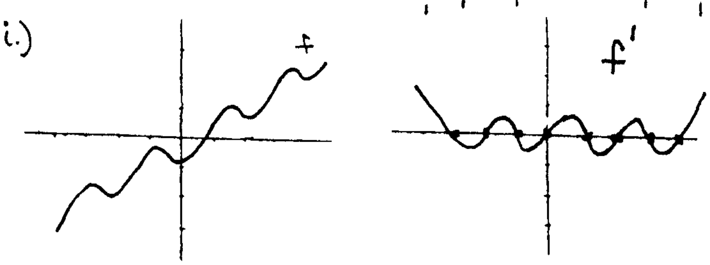
g.)



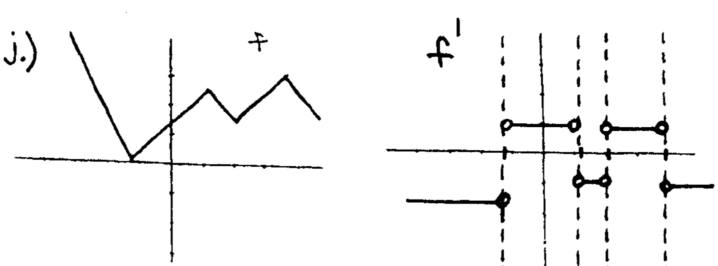
h.)



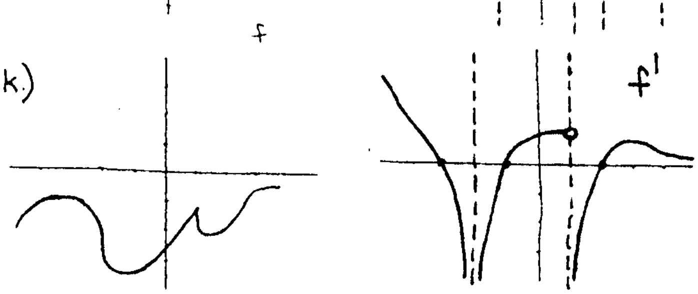
i.)



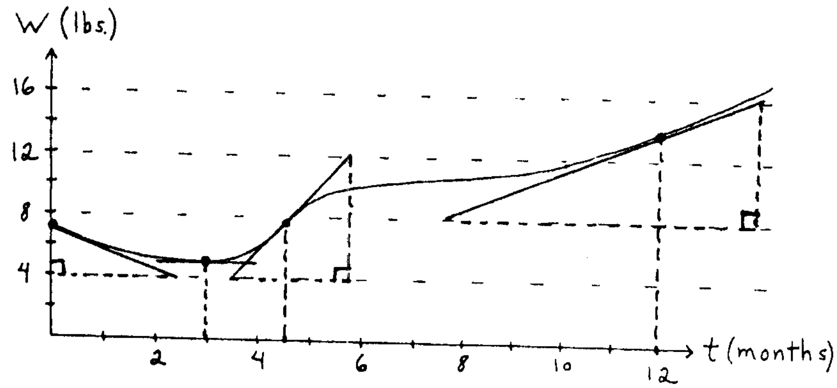
j.)



k.)



2.)



a.) $t=0 \rightarrow W = 7$ lbs. , $t = 3$ mo. $\rightarrow W = 5$ lbs. , $t = 1$ yr. $\rightarrow W = 14$ lbs.

b.) growth rate : slope of tangent line

$t = 0 \rightarrow$ slope $= \frac{-3}{2.5} = -1.2$ lbs./mo. ,

$t = 3$ mo. $\rightarrow$ slope $= 0$ lbs./mo. ,

$t = 1$ yr. $\rightarrow$ slope $= \frac{8}{6.5} = 1.23$ lbs./mo.

c.) The baby is growing at the fastest rate when $t = 4\frac{1}{2}$ months. The growth rate is

$\frac{8}{2.5} = 3.2$ lbs./mo.