Section 3.4

1.) $s = t^2 - 3t + 2 = (t-2)(t-1)$,
vel. $s' = 2t - 3 = 2(t - 3/2)$,
acc. $s'' = 2$; $0 \le t \le 2$

a.) displ. $= s(2) - s(0) = 0 - 2 = -2$ m.,
ave. vel. $= \dfrac{s(2) - s(0)}{2 - 0} = \dfrac{-2}{2} = -1$ m./s.

b.) speed $= |s'(0)| = |-3| = 3$ m./s.,
speed $= |s'(2)| = |1| = 1$ m./s.,
acc. $= s''(0) = 2$ m./s.^{2},
acc. $= s''(2) = 2$ m./s.^{2}

c.) $s'(t) = 2(t - 3/2) = 0 \longrightarrow$
vel. $s' = 0$ m./s. when $\boxed{t = 3/2}$ sec.
and object changes direction
(moving left to moving right)

4.) $s = \frac{1}{4}t^4 - t^3 + t^2$
$= \frac{1}{4}t^2(t^2 - 4t + 4) = \frac{1}{4}t^2(t-2)^2$,
vel. $s' = t^3 - 3t^2 + 2t = t(t-1)(t-2)$,
acc. $s'' = 3t^2 - 6t + 2$; $0 \le t \le 3$

a.) displ. $= s(3) - s(0) = \frac{9}{4} - 0 = \frac{9}{4}$ m.,
ave. vel. $= \dfrac{s(3) - s(0)}{3 - 0} = \dfrac{9/4}{3/1} = \frac{9}{4} \cdot \frac{1}{3} = \frac{3}{4}$ m./s.

b.) speed $= |s'(0)| = |0| = 0$ m./s.

speed $= |s'(3)| = |6| = 6$ m./s.,
acc $= \mp s''(0) = 2$ m./s.^{2},
acc $= s''(3) = 11$ m./s.^{2}

c.) vel. $s' = t(t-1)(t-2) = 0 \rightarrow$
$t = 0$, $t = 1$, or $t = 2$ sec.; when $\boxed{t = 1}$ sec. $s'(1) = 0$ and object changes direction (moving right to moving left); when $\boxed{t = 2}$ sec. $s'(2) = 0$ and object changes direction (moving left to moving right)

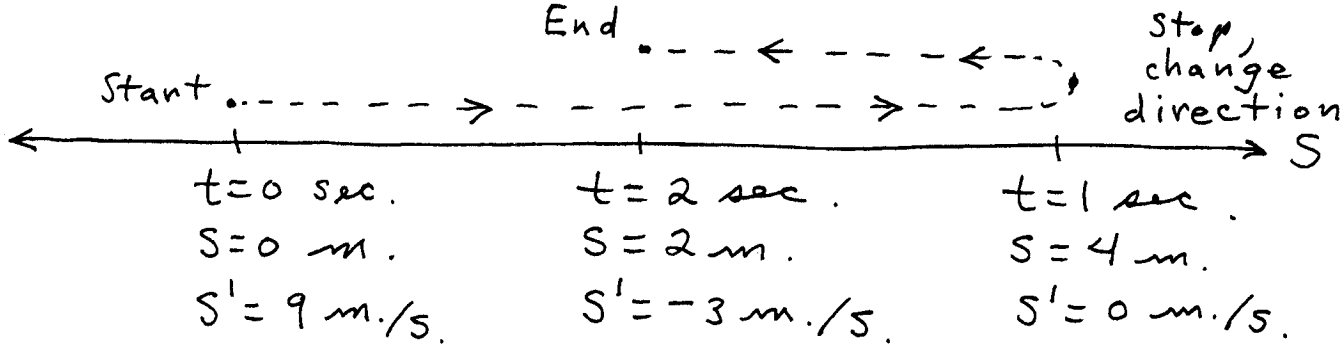
7.) $s = t^3 - 6t^2 + 9t = t(t-3)^2$,
vel. $s' = 3t^2 - 12t + 9$
$= 3(t^2 - 4t + 3) = 3(t-1)(t-3)$,
acc. $s'' = 6t - 12 = 6(t-2)$

a.) vel. $s' = 0 \rightarrow 3(t-1)(t-3) = 0 \rightarrow$
$\boxed{t = 1}$ sec., $\boxed{t = 3}$ sec.;
acc. $s''(1) = -6$ m./s.^{2},
acc. $s''(3) = 6$ m./s.^{2}

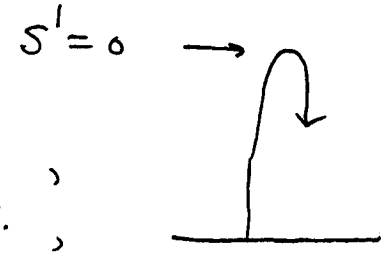
b.) acc. $s'' = 0 \rightarrow 6(t-2) = 0 \rightarrow \boxed{t = 2}$ sec.
$\rightarrow$ vel. $s'(2) = -3$ m./s.

c.) $s(0) = 0$ m. and $s'(0) = 9$ m./s.

so object starts at the origin and begins moving to the right;



the total distance traveled is
$4+2=6$ m.



10.) a.) $S = 24t - 0.8t^2$ m.
vel. $S' = 24 - 1.6t$ m./s.
acc. $S'' = -1.6$ m./s^2

b.) highest point: $S'=0 \rightarrow$
$24 - 1.6t = 0 \rightarrow t = 15$ sec.

c.) $S(15) = 24(15) - 0.8(15)^2 = 180$ m.

d.) If $S = 90$ m. $\rightarrow 90 = 24t - \frac{4}{5}t^2 \rightarrow$
$\frac{4}{5}t^2 - 24t + 90 = 0 \rightarrow 4t^2 - 120t + 450 = 0$
$\rightarrow 2(2t^2 - 60t + 225) = 0 \rightarrow$
$t = \frac{60 \pm \sqrt{3600 - 4(2)(225)}}{4} \approx \frac{60 \pm 42.4}{4} \rightarrow$
$\boxed{t \approx 4.4}$ sec. or $t \approx 25.6$ sec. (NO)

e.) $S = 0 \rightarrow 24t - 0.8t^2 = 0 \rightarrow$

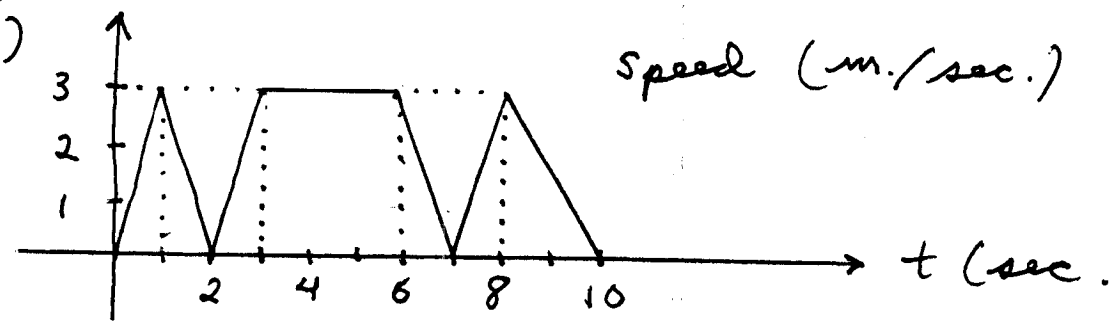
$t(24 - 0.8t) = 0 \rightarrow t = 0$ sec. or
$24 - 0.8t = 0 \rightarrow$ rock is aloft for
$\boxed{t = 30}$ sec.

12.) $s = 832t - 2.6t^2$ (moon)
$h = 832t - 16t^2$ (earth)

a.) $s = 0 \rightarrow 832t - 2.6t^2 = 0 \rightarrow$
$t(832 - 2.6t) = 0 \rightarrow t = 0$ or
$832 - 2.6t = 0 \rightarrow$ bullet is aloft for
$\boxed{t = 320}$ sec. ; $s' = 0 \rightarrow 832 - 5.2t = 0$
$\rightarrow t = 160$ sec. $\rightarrow$ height of bullet
is $s(160) = \underline{66,560 \text{ ft}}$.

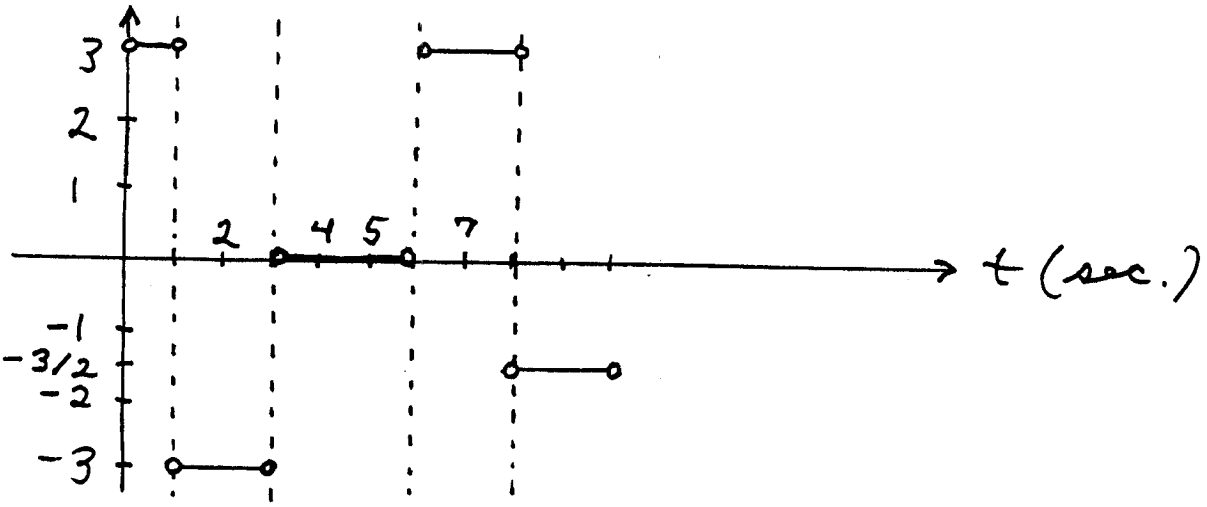
b.) $h = 0 \rightarrow 832t - 16t^2 = 0 \rightarrow$
$t(832 - 16t) = 0 \rightarrow t = 0$ or
$832 - 16t = 0 \rightarrow$ bullet is aloft for
$\boxed{t = 52}$ sec. ; $h' = 0 \rightarrow 832 - 32t = 0$
$\rightarrow t = 26$ sec. $\rightarrow$ height of bullet
is $h(26) = \underline{10,816 \text{ ft}}$.

15.) a.) $t = 2$ sec., $t = 7$ sec.
b.) $3 \le t \le 6$ sec.
c.)

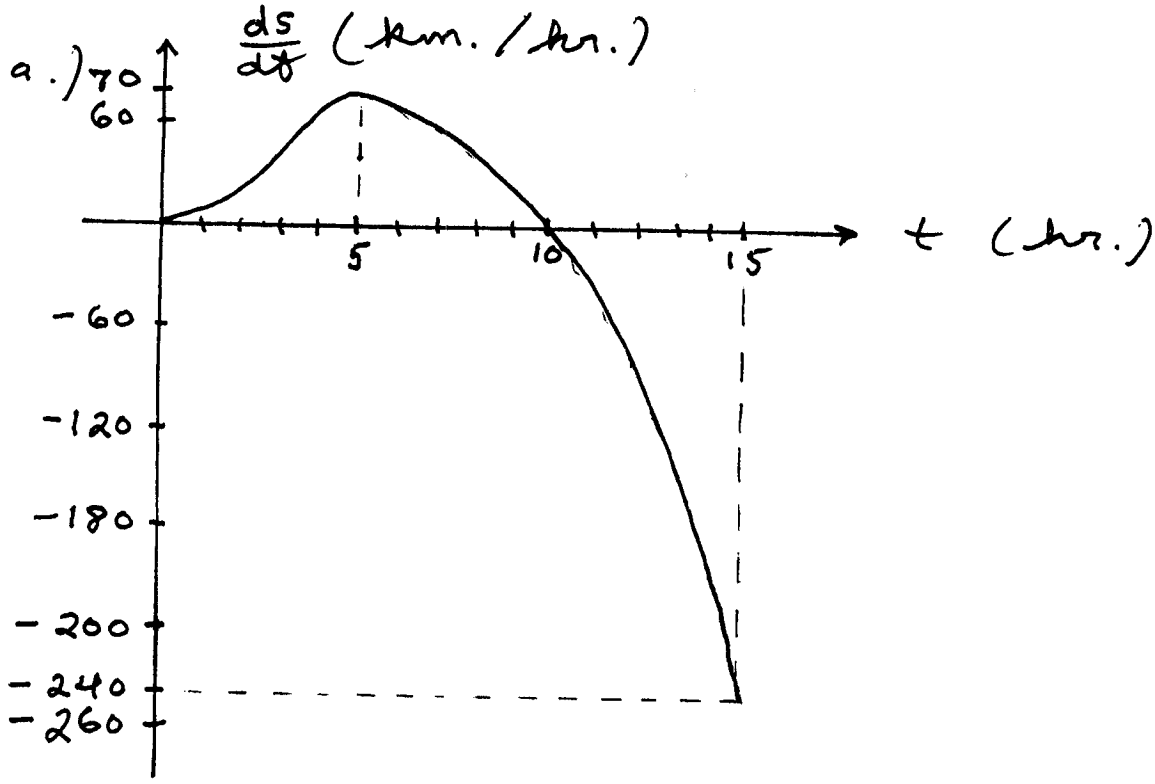


accel. : $\frac{dv}{dt}$ (m./sec.^{2})

d.)



20.) a.)



26.) $Q(t) = 200(30-t)^2 = 200(t^2-60t+900)$

D → $\frac{dQ}{dt} = 200(2t-60)$ and $t = 10$ min. →

$\frac{dQ}{dt} = 200(2(10)-60) = -8000$ gal./min.;

Average rate $= \frac{Q(10)-Q(0)}{10-0} = \frac{80,000-180,000}{10}$

$= -10,000$ gal./min.

29.) Distance $D = \frac{10}{9}t^2$ m. $\longrightarrow$

Velocity $V = \frac{dD}{dt} = \frac{20}{9}t$ m./sec. ;

if $V = \frac{200 \text{ km.}}{1 \text{ hr.}} \cdot \frac{1 \text{ hr.}}{3600 \text{ sec.}} \cdot \frac{1000 \text{ m.}}{1 \text{ km}}$

$= \frac{500}{9}$ m./sec. , then

$\frac{500}{9} = \frac{20}{9}t \longrightarrow \boxed{t = 25}$ sec. and

$D = \frac{10}{9}(25)^2 \approx 694.4$ m.