Section 2.1

1.) a.) $\lim_{x \to 1} g(x)$ DNE since

$\lim_{x \to 1^+} g(x) = 0$ and $\lim_{x \to 1^-} g(x) = 1$.

b.) $\lim_{x \to 2} g(x) = 1$

c.) $\lim_{x \to 3} g(x) = 0$

2.) a.) $\lim_{t \to -2} f(t) = 0$

b.) $\lim_{t \to -1} f(t) = -1$

c.) $\lim_{t \to 0} f(t)$ DNE since

$\lim_{t \to 0^+} f(t) = 1$ and $\lim_{t \to 0^-} f(t) = -1$

3.) a.) T

b.) T

c.) F

d.) F

e.) F

f.) T

5.) <u>Recall</u>: $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$;

$$\lim_{x \to 0^+} \frac{x}{|x|} = \lim_{x \to 0^+} \frac{x}{x} = \lim_{x \to 0^+} 1 = 1;$$

$$\lim_{x \to 0^-} \frac{x}{|x|} = \lim_{x \to 0^-} \frac{x}{-x} = \lim_{x \to 0^-} -1 = -1;$$

so $\lim_{x \to 0} \frac{x}{|x|}$ DNE.

24.) $$\lim_{x \to 1} \frac{-1}{3x-1} = \frac{-1}{3(1)-1} = \frac{-1}{2}$$

26.) $$\lim_{x \to -1} \frac{3x^2}{2x-1} = \frac{3(-1)^2}{2(-1)-1} = \frac{3}{-3} = -1$$

27.) $$\lim_{x \to \frac{\pi}{2}} x \sin x = \frac{\pi}{2} \cdot \sin \frac{\pi}{2} = \frac{\pi}{2}(1) = \frac{\pi}{2}$$

30.) $g(x) = x^2$

a.) $$ARC = \frac{g(1) - g(-1)}{1-(-1)} = \frac{1-1}{2} = \frac{0}{2} = 0$$

b.) $$ARC = \frac{g(0) - g(-2)}{0-(-2)} = \frac{0-4}{0+2} = -2$$

32.) $g(t) = 2 + \cos t$

a.) $$ARC = \frac{g(\pi) - g(0)}{\pi - 0} = \frac{(2+\cos\pi) - (2+\cos 0)}{\pi}$$

$$= \frac{2+(-1)-2-(1)}{\pi} = \frac{-2}{\pi}$$

b.) $ARC = \dfrac{g(\pi)-g(-\pi)}{\pi-(-\pi)}$

$$= \frac{(2+\cos\pi)-(2+\cos(-\pi))}{2\pi}$$

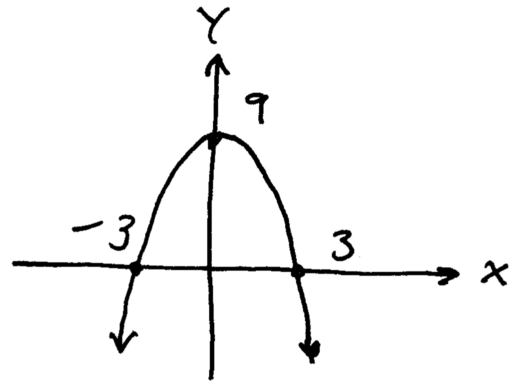
$$= \frac{2+(-1)-2-(-1)}{2\pi} = \frac{0}{2\pi} = 0$$

33.) $R(\theta) = \sqrt{4\theta+1}$

$$ARC = \frac{R(2)-R(0)}{2-0} = \frac{\sqrt{9}-\sqrt{1}}{2} = \frac{3-1}{2} = 1$$

Worksheet 1

1.) a.) $f(x) = 9 - x^2$
(parabola)

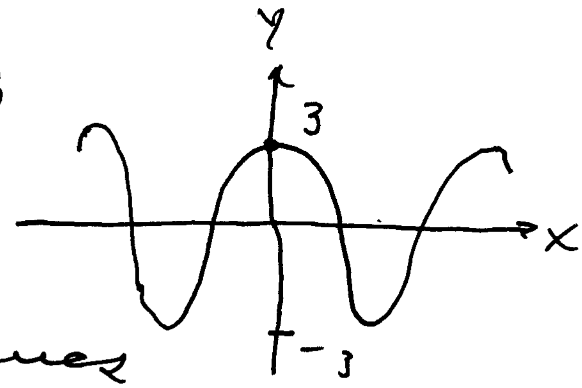


Domain : all x-values
Range : $Y \leq 9$

b.) $f(x) = 3 \cos 4x$;

$-1 \leq \cos 4x \leq +1 \rightarrow$

$-3 \leq 3 \cos 4x \leq 3$;



Domain : all x-values
Range : $-3 \leq Y \leq +3$

c.) $y = 5 + \sqrt{16 - x}$; $16 - x \geq 0 \rightarrow$
$x \leq 16$ so

Domain : $x \leq 16$;

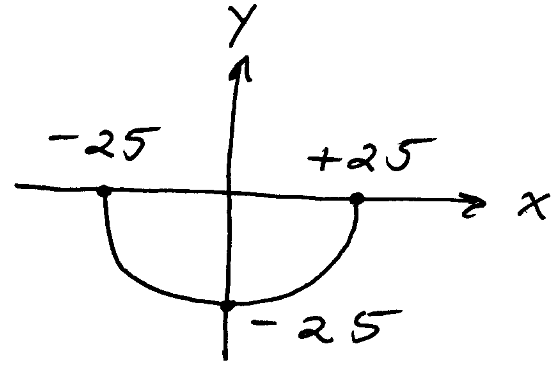
$0 \leq \sqrt{16-x} < \infty$ so
$5 \leq 5 + \sqrt{16 - x} < \infty$,

Range : $y \geq 5$

d.) $y = -\sqrt{625 - x^2} \rightarrow$

$y^2 = 625 - x^2 \rightarrow x^2 + y^2 = 25^2$
(circle centered at $(0,0)$
with radius $r = 25$) ; so

$Y = -\sqrt{625 - x^2}$
is bottom half
of circle ;



<u>Domain</u> : $-25 \le x \le 25$

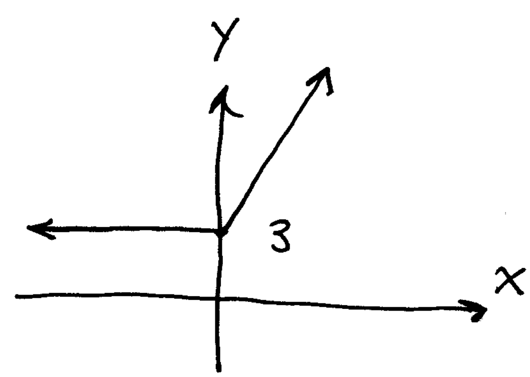
<u>Range</u> : $-25 \le Y \le 0$

e.) <u>Recall</u> : $|x| = \begin{cases} x & \text{if } x \ge 0 \\ -x & \text{if } x < 0 \end{cases}$;

$y = x + |x| + 3$

$= \begin{cases} x + (x) + 3 & \text{if } x \ge 0 \\ x + (-x) + 3 & \text{if } x < 0 \end{cases}$

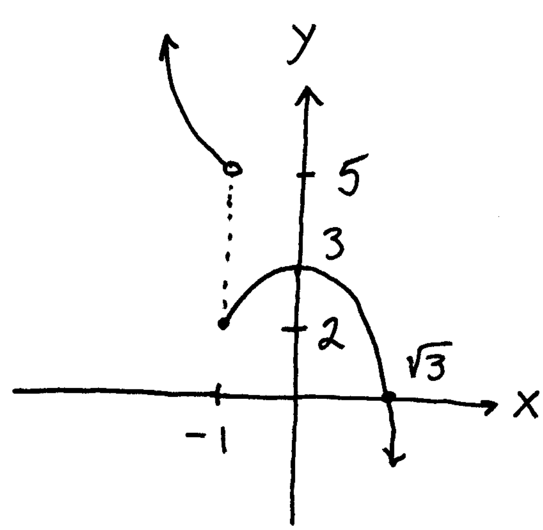
$= \begin{cases} 2x + 3 & \text{if } x \ge 0 \\ 3 & \text{if } x < 0 \end{cases}$



<u>Domain</u> : all x-values

<u>Range</u> : $Y \ge 3$

f.) $f(x) = \begin{cases} x^2 + 4 & \text{if } x < -1 \\ 3 - x^2 & \text{if } x \ge -1 \end{cases}$



<u>Domain</u> : all x-values

<u>Range</u> : $Y \le 3$, $Y > 5$

2.) $f(x) = \dfrac{7}{3-\sqrt{x^2-16}}$;

$x^2 - 16 = (x-4)(x+4) \geq 0$

sign chart : $\begin{array}{ccccc} + & 0 & - & 0 & + \\ \hline & x=-4 & & x=4 & \end{array}$

so $x \geq 4$, $x \leq -4$; <u>AND</u>

$3 - \sqrt{x^2-16} \neq 0 \longrightarrow 3 \neq \sqrt{x^2-16} \longrightarrow$

$9 \neq x^2 - 16 \longrightarrow x^2 \neq 25 \longrightarrow x \neq \pm 5$;

<u>Domain</u> : $x \geq 4, x \leq -4$ and $x \neq \pm 5$

3.) $f(x) = \dfrac{x}{3-x}$, $g(x) = \dfrac{x-2}{x+4}$

a.) $(f \circ g)(x) = f(g(x))$

$$= f\left(\frac{x-2}{x+4}\right) = \frac{\left(\frac{x-2}{x+4}\right)}{3-\left(\frac{x-2}{x+4}\right)} \cdot \frac{x+4}{x+4}$$

$$= \frac{x-2}{3(x+4)-(x-2)}$$

$$= \frac{x-2}{3x+12-x+2} = \frac{x-2}{2x+14}$$

b.) $(g \circ f)(x) = g(f(x))$

$$= g\left(\frac{x}{3-x}\right) = \frac{\left(\frac{x}{3-x}\right)-2}{\left(\frac{x}{3-x}\right)+4} \cdot \frac{3-x}{3-x}$$

$$= \frac{x - 2(3-x)}{x + 4(3-x)} = \frac{x - 6 + 2x}{x + 12 - 4x} = \frac{3x - 6}{12 - 3x}$$

$$= \frac{3(x-2)}{3(4-x)} = \frac{x-2}{4-x}$$