

## Section 2.2

$$7.) \lim_{x \rightarrow 2} \frac{x+3}{x+6} = \frac{5}{8}$$

$$15.) \lim_{h \rightarrow 0} \frac{3}{\sqrt{3h+1} + 1} = \frac{3}{\sqrt{1} + 1} = \frac{3}{1+1} = \frac{3}{2}$$

$$18.) \lim_{h \rightarrow 0} \frac{\sqrt{5h+4} - 2}{h} \stackrel{\frac{0}{0}}{=} \lim_{h \rightarrow 0} \frac{\sqrt{5h+4} - 2}{h} \cdot \frac{\sqrt{5h+4} + 2}{\sqrt{5h+4} + 2}$$

$$= \lim_{h \rightarrow 0} \frac{(5h+4) - 4}{h(\sqrt{5h+4} + 2)} = \lim_{h \rightarrow 0} \frac{5h}{h(\sqrt{5h+4} + 2)}$$

$$= \frac{5}{2+2} = \frac{5}{4}$$

$$20.) \lim_{x \rightarrow -3} \frac{x+3}{x^2+4x+3} \stackrel{\frac{0}{0}}{=} \lim_{x \rightarrow -3} \frac{x+3}{(x+3)(x+1)}$$

$$= \lim_{x \rightarrow -3} \frac{1}{x+1} = \frac{1}{-2} = -\frac{1}{2}$$

$$23.) \lim_{t \rightarrow 1} \frac{t^2+t-2}{t^2-1} \stackrel{\frac{0}{0}}{=} \lim_{t \rightarrow 1} \frac{(t-1)(t+2)}{(t-1)(t+1)} = \frac{3}{2}$$

$$26.) \lim_{y \rightarrow 0} \frac{5y^3+8y^2}{3y^4-16y^2} = \lim_{y \rightarrow 0} \frac{y^2(5y+8)}{y^2(3y^2-16)}$$

$$= \frac{8}{-16} = -\frac{1}{2}$$

$$27.) \lim_{u \rightarrow 1} \frac{u^4-1}{u^3-1} \stackrel{\frac{0}{0}}{=} \lim_{u \rightarrow 1} \frac{(u^2-1)(u^2+1)}{(u-1)(u^2+u+1)}$$

$$= \lim_{u \rightarrow 1} \frac{(u-1)(u+1)(u^2+1)}{(u-1)(u^2+u+1)} = \frac{(2)(2)}{3} = \frac{4}{3}$$

$$\begin{aligned}
 31.) \lim_{x \rightarrow 1} \frac{x-1}{\sqrt{x+3}-2} &\stackrel{\frac{0}{0}}{=} \lim_{x \rightarrow 1} \frac{x-1}{\sqrt{x+3}-2} \cdot \frac{\sqrt{x+3}+2}{\sqrt{x+3}+2} \\
 &= \lim_{x \rightarrow 1} \frac{(x-1)(\sqrt{x+3}+2)}{(x+3)-4} = \lim_{x \rightarrow 1} \frac{(x-1)(\sqrt{x+3}+2)}{\cancel{x-1}} \\
 &= 2+2 = 4
 \end{aligned}$$

$$\begin{aligned}
 34.) \lim_{x \rightarrow -2} \frac{x+2}{\sqrt{x^2+5}-3} &\stackrel{\frac{0}{0}}{=} \lim_{x \rightarrow -2} \frac{x+2}{\sqrt{x^2+5}-3} \cdot \frac{\sqrt{x^2+5}+3}{\sqrt{x^2+5}+3} \\
 &= \lim_{x \rightarrow -2} \frac{(x+2)(\sqrt{x^2+5}+3)}{(x^2+5)-9} \\
 &= \lim_{x \rightarrow -2} \frac{(x+2)(\sqrt{x^2+5}+3)}{x^2-4} \\
 &= \lim_{x \rightarrow -2} \frac{\cancel{(x+2)}(\sqrt{x^2+5}+3)}{\cancel{(x+2)}(x-2)} \\
 &= \frac{3+3}{-4} = -\frac{3}{2}
 \end{aligned}$$

$$\begin{aligned}
 39.) \text{ a.) } \lim_{x \rightarrow c} f(x)g(x) &= \lim_{x \rightarrow c} f(x) \cdot \lim_{x \rightarrow c} g(x) \\
 &= (5)(-2) = -10
 \end{aligned}$$

$$\begin{aligned}
 \text{ b.) } \lim_{x \rightarrow c} 2f(x)g(x) &= 2 \cdot \lim_{x \rightarrow c} f(x) \cdot \lim_{x \rightarrow c} g(x) \\
 &= 2(5)(-2) = -20
 \end{aligned}$$

$$\begin{aligned}
 \text{ c.) } \lim_{x \rightarrow c} (f(x)+3g(x)) \\
 &= \lim_{x \rightarrow c} f(x) + 3 \cdot \lim_{x \rightarrow c} g(x) = 5 + 3(-2) = -1
 \end{aligned}$$

$$d.) \lim_{x \rightarrow c} \frac{f(x)}{f(x) - g(x)} = \frac{\lim_{x \rightarrow c} f(x)}{\lim_{x \rightarrow c} f(x) - \lim_{x \rightarrow c} g(x)}$$

$$= \frac{5}{5 - (-2)} = \frac{5}{7}$$

50.)  $2 - x^2 \leq g(x) \leq 2 \cos x$ , then

$$\lim_{x \rightarrow 0} (2 - x^2) = 2 - 0 = 2 \text{ and}$$

$$\lim_{x \rightarrow 0} 2 \cos x = 2 \cos 0 = 2 \cdot (1) = 2, \text{ so}$$

by Squeeze Principle  $\lim_{x \rightarrow 0} g(x) = 2$

51.) a.)  $1 - \frac{x^2}{6} \leq \frac{x \sin x}{2 - 2 \cos x} \leq 1$ , then

$$\lim_{x \rightarrow 0} \left(1 - \frac{x^2}{6}\right) = 1 - 0 = 1 \text{ and}$$

$$\lim_{x \rightarrow 0} 1 = 1, \text{ so by Squeeze Principle}$$

$$\lim_{x \rightarrow 0} \frac{x \sin x}{2 - 2 \cos x} = 1.$$

56.)  $\lim_{x \rightarrow -2} \frac{f(x)}{x^2} = 1 \rightarrow \frac{\lim_{x \rightarrow -2} f(x)}{\lim_{x \rightarrow -2} x^2} = 1 \rightarrow$

$$\frac{\lim_{x \rightarrow -2} f(x)}{4} = 1 \rightarrow \lim_{x \rightarrow -2} f(x) = 4$$

a.)  $\lim_{x \rightarrow -2} f(x) = 4$

b.)  $\lim_{x \rightarrow -2} \frac{f(x)}{x} = \frac{\lim_{x \rightarrow -2} f(x)}{\lim_{x \rightarrow -2} x} = \frac{4}{-2} = -2$

$$57.) a.) \lim_{x \rightarrow 2} \frac{f(x) - 5}{x - 2} = 3 \rightarrow$$

$$\frac{\lim_{x \rightarrow 2} (f(x) - 5)}{\lim_{x \rightarrow 2} (x - 2)} = 3 \rightarrow \frac{\lim_{x \rightarrow 2} (f(x) - 5)}{0} = 3$$

$$\rightarrow \lim_{x \rightarrow 2} (f(x) - 5) = 0 \rightarrow \lim_{x \rightarrow 2} f(x) - \lim_{x \rightarrow 2} 5 = 0$$

$$\rightarrow \lim_{x \rightarrow 2} f(x) - 5 = 0 \rightarrow \boxed{\lim_{x \rightarrow 2} f(x) = 5}$$

(If  $\lim_{x \rightarrow 2} (f(x) - 5) = k \neq 0$ , then

$$\frac{\lim_{x \rightarrow 2} (f(x) - 5)}{0} = \frac{k}{0} = \pm \infty.)$$

$$58.) \lim_{x \rightarrow 0} \frac{f(x)}{x^2} = 1 \rightarrow \frac{\lim_{x \rightarrow 0} f(x)}{\lim_{x \rightarrow 0} x^2} = 1 \rightarrow$$

$$\frac{\lim_{x \rightarrow 0} f(x)}{0} = 1 \rightarrow \lim_{x \rightarrow 0} f(x) = 0$$

$$a.) \lim_{x \rightarrow 0} f(x) = 0$$

$$b.) \frac{f(x)}{x} = \frac{f(x) \cdot x}{x^2} = \frac{f(x)}{x^2} \cdot x \rightarrow$$

$$\lim_{x \rightarrow 0} \frac{f(x)}{x} = \lim_{x \rightarrow 0} \frac{f(x)}{x^2} \cdot \lim_{x \rightarrow 0} x = (1) \cdot (0) = 0.$$